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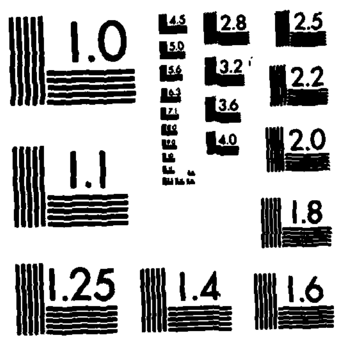

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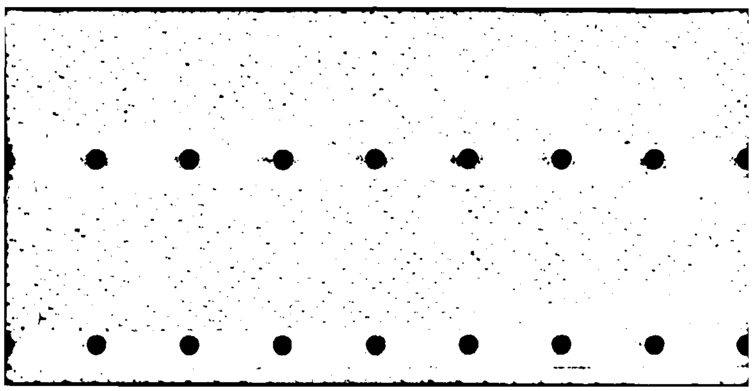

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A NEW GOODNESS OF FIT TEST FOR
THE UNIFORM DISTRIBUTION WITH
UNSPECIFIED PARAMETERS

THESIS

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A NEW GOODNESS OF FIT TEST FOR THE UNIFORM DISTRIBUTION WITH UNSPECIFIED PARAMETERS

THESIS

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—Larry B. Woodbury

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Abstract

Separate techniques of interpolation, reflection, and parameter estimation are combined to develop a new goodness of fit test for the uniform distribution. The Kolmogorov-Smirnov, Anderson-Darling, and Cramer-Von Mises statistics are used in the generation of critical value tables of sample sizes from 3 to 50. The methods for estimating parameters are the Maximum Likelihood and the Best Linear Unbiased Estimators. Separate tables for each are presented. These tables are built with and without employing the reflection technique. The reflection technique is one in which the data points are reflected about the sample mean to double the size of the sample set.

With these tables of critical values, a power study is done to test the power of the three statistics with the reflection procedure versus the same statistics without the reflection procedure. The powers are generally higher for the statistics modified with the reflection procedure; however, they are found to be smaller for data distributions that are non-symmetrical or Cauchy. The power for the Anderson-Darling statistic using the Maximum Likelihood Estimators is found to be of little value while the powers of all statistics were found to be improved by using the Best Linear Unbiased Estimators instead of the Maximum Likelihood Estimators.

A NEW GOODNESS OF FIT TEST FOR THE UNIFORM DISTRIBUTION WITH UNSPECIFIED PARAMETERS

I. Introduction

The concept of goodness of fit has been used for some time in statistical analysis. This concept is based on the assumption that one can take a set of data and perform tests on that data to see if it fits (or corresponds to) a known probability distribution. This involves using statistical tests with the following hypotheses:

H_0: $f(x)$ = known probability distribution

H_A: $f(x) \neq$ known probability distribution

where

H_0 is the null hypothesis

H_A is the alternate hypothesis

$f(x)$ is the assumed distribution that the analyst thinks the data fit

The analyst usually wants to be able to accept the null hypothesis (H_0) that the distribution of the data is known.

This type of testing involves a yes/no decision. Either the data fit the hypothesized distribution well enough to be able to assign the associated properties of the distribution to the data, or the data don't fit (Ref 15:2). In this yes or no decision, which is based on certain fixed criteria, there is no way to tell how good the fit really is without referring to other studies. The only conclusion that is drawn is that the null hypothesis is either accepted or rejected.

Power Concept in Goodness of Fit Tests

A few relatively recent studies (Refs 9; 15; 19) have applied certain individual techniques that will be combined in this thesis to support the concept of power as a measure of goodness of fit. These techniques will be described in later sections.

The power of a test is the probability of rejecting the null hypothesis (H_0) when the alternate hypothesis (H_A) is true (Ref 14:403-404). Thus, the power becomes a statement of confidence in the ability to reject the null hypothesis when the alternate hypothesis is in fact true. To give an example, suppose a data sample is drawn. Now suppose that the involved null hypothesis, H_0, is that the data come from a uniform distribution and the alternate hypothesis, H_A, is that the data are not uniform. A goodness of fit test is run and the calculated statistic is found to be .360 at $\alpha = .01$. From a table of critical values it is found that .462 is the critical value for that statistic. Since the calculated statistic is less than the critical value, the null hypothesis would fail to be rejected. Now suppose that a power study for that statistic is looked at and it is found that the power versus a normal distribution is .98. The conclusion that the data are not normal, but uniform could be stated with high confidence. If another power study was considered where the power versus the Cauchy distribution was found to be .23, not much confidence could be placed in the conclusion that the data are not Cauchy, but uniform (Ref 15:3-4).

Problems with Goodness of Fit Statistics and Associated Power Studies

When using many of the goodness of fit statistics, one should be aware of some of the problems that are associated with them. One of these problems is that with small sample sizes, and at low significance levels (alpha levels), these statistics are not very powerful. That is, they have relatively low power values which means there is a good possibility that the null hypothesis is not being rejected when the alternate hypothesis is true. This problem is evident for all test statistics concerned within this thesis.

A second problem is that the method of estimating parameters where those parameters are unknown has a great effect on both critical values in the rejection tables and on the power values. This thesis looks at two methods for estimating the unknown parameters. The first method is the Maximum Likelihood Estimator method and the second is the Best Linear Unbiased Estimator method. The results of these two methods are very different and are discussed in later chapters.

A third problem that is encountered in goodness of fit tests is that the power of the tests vary with the distributions involved. Because of this the power study becomes important and useful to the analyst (Ref 15:3).

Statistics and Techniques Used in Thesis

Besides the two techniques for estimating parameters already mentioned, there are two other important techniques and three statistics involved in this thesis that are presented in this section.

Three Test Statistics. There are several test statistics that are used in goodness of fit hypothesis testing. Of these, the three that are considered in this research are the Kolmogorov-Smirnov (K-S), the Anderson-Darling (A-D), and the Cramer-Von Mises (CVM) test statistics. These are standard tests and are used to test whether or not a set of data comes from a completely specified distribution. However, when the parameters are unknown, they must be estimated and the test statistics modified accordingly. This has been done in several cases of distributions with unknown parameters. For example, H. W. Lilliefors did it for the normal distribution and the K-S test (Ref 12), R. Cortes did it for the Gamma and

Weibull distributions and the K-S test (Ref 4), and J. Green and Y. Hegazy did it for all three tests (K-S, A-D, and CVM) and for several distributions among which was the uniform distribution (Ref 7). Since this thesis is concerned with the uniform distribution, the work by Green and Hegazy will be referenced and some of their methodology will be followed.

Two Important Techniques. This thesis uses two important techniques that have a direct bearing on the thesis objectives. The first is a reflection technique that was developed by E. F. Schuster in 1973 (Ref 19) and subsequently used in a study of the normal distribution by Thomas J. Ream in 1981 (Ref 15). The second technique is one for representing ordered statistics on a continuous spectrum which allows interpolation and extrapolation of critical values. This technique is known as the Bootstrap technique and was developed by B. Efron (Ref 6) and has been used in studies by J. Johnston (Ref 9) and also by Thomas Ream (Ref 15). These techniques will be used in a similar manner in this research.

Reflection Technique. The concept of reflection is a main part of this thesis. It takes the ordered data in the sample set and reflects it about the sample mean so that a symmetric sample that is twice the size of the original sample set is produced. It is hoped that this increase in size will cause an increase in the power of the calculated statistic.

Bootstrap Technique. The boostrap technique is used in this thesis in the same manner as it was used by Johnston (Ref 9) and Ream (Ref 15). It pertains to the generation of critical value tables (also known as rejection tables). The critical value tables are usually developed by randomly calculating very large numbers of the statistic involved and then ordering those values of the statistic. The critical value for a given significance level would simply be the statistic that corresponds to the percentage point for that significance level. For example, suppose 5000 statistics are calculated and ordered and the significance level is set at .10 (α = .10). The critical value would simply be the statistic that corresponds to the 90 percent level or, in other words, the 4500th ordered statistic. The bootstrap technique takes these ordered, discrete statistics and plots them on a continuous spectrum between zero and one. When this is done the spaces between each of the plotted statistics represent piece-wise linear functions and this makes it possible to interpolate for the desired percentage level and thus a more accurate value can be obtained.

Primary Thesis Objectives

There are three primary objectives in this research effort concerning the uniform distribution. The first is to generate tables of critical values for the Kolmogorov-Smirnov, Anderson-Darling, and Cramer-Von Mises test statistics using the Schuster concept of reflection of data points about the sample mean. Also, a set of these tables will be developed for these same test statistics when the data points are not reflected. It is noted here that Green and Hegazy developed tables for critical values for the uniform distribution with unspecified parameters using these three test statistics but their's was based on a different number of statistic calculations. They also did not use the interpolation technique (bootstrap) nor were their data sample sizes extensive. Because of these differences in methodology, the new tables were needed and there will be some differences observed.

The second objective is to perform a power study using the above-mentioned tables and to determine if there is an increase in power of the goodness of fit test when the reflection technique is used.

The third objective is to conduct the study using both the maximum likelihood estimators and the best linear unbiased estimators to compare the powers for each to determine which is the best to use.

Format of Thesis

This thesis is composed of five chapters. This introduction is the first chapter and is meant to tell what the research is about. Chapter II gives a background of the uniform distribution and more details about the techniques that are used in the study. Chapter III is the procedure chapter. The procedures followed in the study as well as the methodology are outlined here. Chapter IV provides the results of the power study and Chapter V contains the conclusions and recommendations. The appendices contain the power value tables and examples of computer programs used in the calculation of the critical value tables and in the generation of the power tables.

II. Background

The purpose of this chapter is to explain in further detail the techniques and test statistics that are used in this thesis. The format of this chapter is as follows:

1. The uniform distribution along with the two methods for estimating parameters is briefly discussed.

2. The reflection technique is then explained in further detail with an accompanying example.

3. The bootstrap technique is the next topic to be covered. A statement of the plotting position is included here.

4. The three test statistics (K-S, A-D, and CVM) are outlined and examples of each are presented.

Uniform Distribution

A random variable that covers an interval (a,b) is said to be uniform if it has a probability density function (pdf) given by:

$$f(x) = \begin{cases} \dfrac{1}{b - a} & \text{if } a < x < b \\ 0 & \text{otherwise} \end{cases} \qquad (1)$$

This pdf is used to calculate the cumulative distribution function (CDF) which is given as follows:

$$F(x) = \begin{cases} 0 & \text{if } x \leq a \\ \dfrac{x-a}{b-a} & \text{if } a < x < b \\ 1 & \text{if } x \geq b \end{cases} \tag{2}$$

These equations are used both in the calculations of the critical value tables and in the power study. The problem here is that when a and b are unknown they must be estimated. As mentioned in Chapter I, this research was accomplished using two methods of estimation, the maximum likelihood estimators (MLE) and the best linear unbiased estimators (BLUE). The MLE for the uniform distribution are found in the following way:

Let $x_1, x_2, x_3, \ldots, x_n$ be a random sample of observations from a uniform distribution with the pdf given in Eq (1) above. Then the likelihood function, L, is the product of the individual pdf values. That is

$$L = f(x_1)f(x_2)\ldots f(x_n) = \left(\frac{1}{b-a}\right)\left(\frac{1}{b-a}\right)\ldots\left(\frac{1}{b-a}\right) \tag{3}$$

$$= \left(\frac{1}{b-a}\right)^n \tag{4}$$

The object is to find a and b that maximizes L above. In the case of the uniform distribution, it is noted that L is a monotonically decreasing function of a and b. Since L increases as (b-a) decreases and since (b-a) must be equal to or greater than the maximum observation in the sample set, the estimated values of a and b that maximize

L are the smallest and largest observation in the sample (Ref 14:365). Therefore, after ordering the sample set of observations, the values of a and b are simply the values of $x_{(1)}$ and $x_{(n)}$ respectively. These MLE for a and b were used in the first phase of the computations.

The BLUE method was the second method for estimating a and b. This method effectively takes the average distance between the ordered sample set of observations and sets the lower bound, $\hat{a}$, that distance below $x_{(1)}$ and sets the upper bound, $\hat{b}$, that same distance above $x_{(n)}$. This is done by the following equations:

$$\hat{a} = x_{(1)} - (x_{(n)} - x_{(1)} / (n - 1) \tag{5}$$

$$\hat{b} = x_{(n)} - (x_{(n)} - x_{(1)} / (n - 1) \tag{6}$$

where

$x_{(1)}$ is the first ordered observation

$x_{(n)}$ is the last or nth ordered observation and

n is the number of observations in the set

As will be seen, this BLUE method for estimating the uniform parameters (Ref 17:387) was found to be of greater utility and gave more meaningful information in the power study than did the MLE.

Reflection Technique

The first step in the reflection technique, as presented in this research, is to order the sample set from the smallest to largest. This is really not necessary if doubling the sample size is the only objective. It is done here for clarity and easier following. The mean is then calculated using the standard uniform equation for the mean

$$\hat{\mu} = \frac{\hat{a} + \hat{b}}{2} \tag{7}$$

The next step is to take each random deviate (observation) in the set and find the point on the opposite side of the mean that is equal distant from the mean as that point. This is done in the following manner:

Let $i = 1, 2, 3, \ldots, n$ where n is the number of observations in the set. Let $\overline{\mu}$ equal the calculated mean (Eq 7) and let $(x_{(1)}, x_{(2)}, x_{(3)}, \ldots, x_{(n)})$ be the ordered set of random observations. For each i we are looking for x_{n+i}, the new reflected point.

$$x_{n+i} = 2\overline{\mu} - x_i \tag{8}$$

It is observed that the original mean (before reflection) is the same mean of the newly formed symmetrical reflected set and each point is the same distance from the mean and its reflected point. Table 1 gives a numerical example of this technique.

Table 1

Reflection of Data Points About the Mean
For a Sample Set n = 5

Data Points	Mean	Reflected Data Points $(2\bar{\mu} - x_i)$	Mean
.174		.864	
.411		.627	
.502	.519	.536	.519
.678		.360	
.864		.174	

Complete Ordered Set Now of Size 10		Mean
.174	.536	
.174	.627	
.360	.678	.519
.411	.864	
.502	.864	

<u>Bootstrap Technique</u>

The main purpose for using the bootstrap technique is to obtain a method for interpolating the critical values at different significance levels. This technique is based on a plotting procedure that plots the calculated critical values on the horizontal axis and calculates a value between zero and one on the vertical axis that corresponds to each of the critical values. There are three plotting position procedures that can be used for calculating the vertical values in the bootstrap technique. These are the median rank, the modified step rank, and the average of the mode and mean ranks. A detailed discussion of these three plotting procedures is not presented here since a complete description is given both by Bush (Ref 3) and by Ream (Ref 15) in their research. It is sufficient to note here that all three of these procedures possess a desired (and required) symmetrical property that enable them to assign positions between zero and one to each of the corresponding critical values. It is also noted that there is very little difference between the values generated by these three procedures when the sample size gets very large. That is, when the number of critical values that are used gets above 50, the differences between the plotting values for the three procedures are only in the fourth decimal place. For this reason the modified step rank procedure is used in this thesis because of its simplicity and ease of programming and computation.

Modified Step Rank. The formula for the step rank is

$$\text{step rank} = \frac{i - 1}{n} \tag{9}$$

As this formula is written, it does not have the necessary symmetry property that is mentioned above. To obtain this symmetry property, Eq 9 is modified as shown in the following equation.

$$\text{modified step rank} = \frac{i - 0.5}{n} \tag{10}$$

This is the plotting procedure used in this thesis for the generation of all the critical value tables. As it will be shown in the next section, it is an integral part of the bootstrap technique.

Example of Bootstrap Technique Using Modified Step Rank. To give an example of how this modified step rank procedure is applied in this study, consider the following:

1. Ten random deviates are obtained from a uniform distribution.

2. With these ten random deviates, a K-S statistic is calculated.

3. These first two steps are repeated ten times to yield ten K-S statistics (different seeds are used for each time).

4. The ten K-S statistics (denoted by X_i, for $i = 1, 2, \ldots, 10$) are then ordered from smallest to largest.

5. The modified step rank (Y_i) is calculated for each i using Eq 10.

A numerical example of these steps is shown in Table 2. The values in Table 2 are the values used in the following example of the bootstrap technique.

To obtain a fully continuous function between zero and one, the extrapolation process is needed to find the end values of this function. These end points are represented by $X_{(0)}$ and $X_{(11)}$. This process uses the standard linear slope-intercept formula

Table 2

K-S Ordered Statistics and Values of the Modified Step Rank for Sample Size of 10

i	Modified Step Rank ($Y_{(i)}$)	Ordered Statistic ($X_{(i)}$)
1	.05	.1663
2	.15	.1696
3	.25	.1839
4	.35	.1859
5	.45	.2004
6	.55	.2195
7	.65	.2252
8	.75	.2333
9	.85	.2868
10	.95	.3389

$$y = mx + b \tag{11}$$

to find the values of X_0 and X_{11}. The values for X_0 and X_{11} are extrapolated critical values that correspond to 0 and 1 on the vertical axis. Using Eq 11 and the above table we find the slope

$$m = \frac{Y_2 - Y_1}{X_{(2)} - X_{(1)}} = \frac{.15 - .05}{.1696 - .1663} = 30.3030 \tag{12}$$

and the intercept

$$\begin{aligned} b = Y_1 - m(X_{(1)}) &= .05 - (30.3030)(.1663) \\ &= -4.9894 \end{aligned} \tag{13}$$

and finally X_0

$$X_{(0)} = \frac{0.0 - b}{m} = \frac{0.0 - (-4.9894)}{30.3030} = .16465 \tag{14}$$

It is noted that in some cases, the critical value at $X_{(0)}$ will turn out to be less than zero. In these cases, $X_{(0)}$ is simply set to zero.

Extrapolation for $X_{(11)}$ is accomplished in the same way as $X_{(0)}$. We first find the slope

$$m = \frac{Y_{10} - Y_9}{X_{(10)} - X_{(9)}} = \frac{.95 - .85}{.3389 - .2868} = 1.9194 \tag{15}$$

and then the intercept

$$\begin{aligned} b = Y_9 - m(X_{(9)}) &= .85 - (1.9194)(.2868) \\ &= .2995 \end{aligned} \tag{16}$$

and X_{11}

$$X_{(11)} = \frac{1.0 - b}{m} = \frac{1.0 - .2995}{1.9194} = .36495 \tag{17}$$

This now yields a completely continuous piece-wise linear function between zero and one. Figure 1 shows how all of these values are correlated in a graph.

Let us now suppose that we want to look at a significance level of .10 ($\alpha = .10$). It is mentioned earlier that Green and Hegazy (Ref 8) as well as others, would simply choose the ninth largest ordered statistic (.2868 from Table 2) as the critical value. However, by using the bootstrap method, a more accurate critical value is .3129. This is calculated as follows:

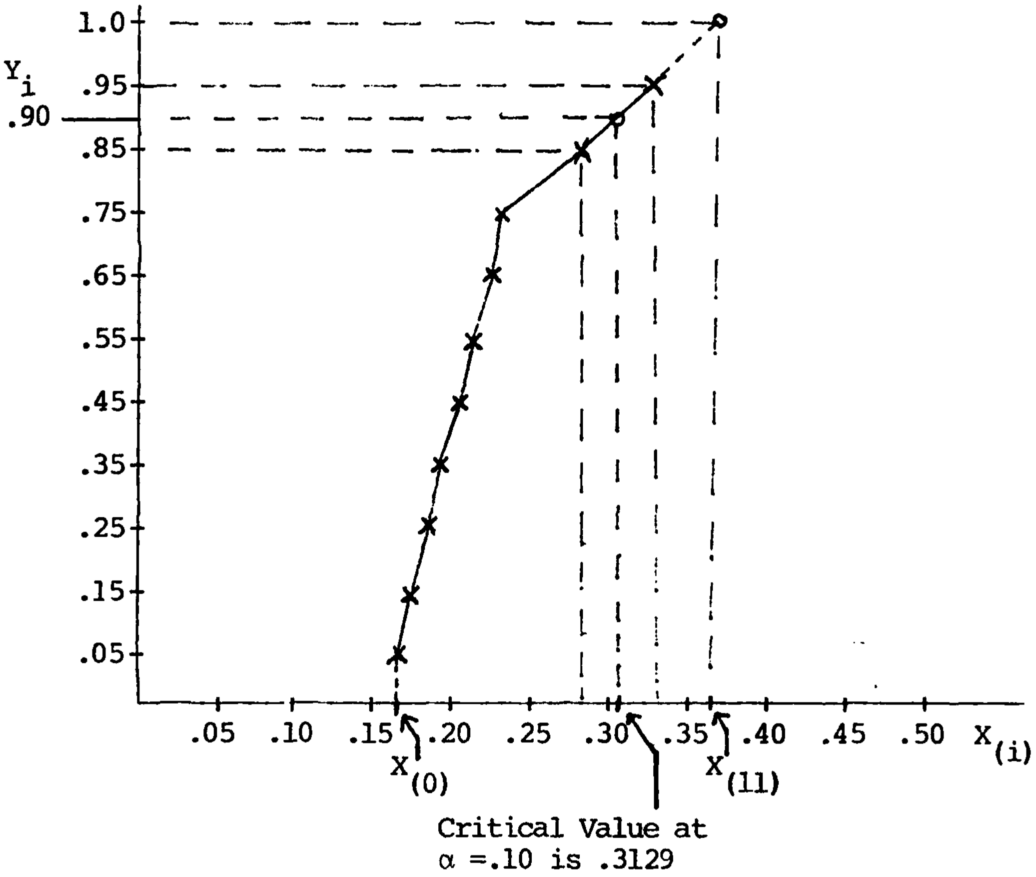

Figure 1. Example of Bootstrap Technique Using Modified Step Rank

First,

$$m = 1.9194 \text{ from Eq 15.}$$

Next,

$$b = .2995 \text{ from Eq 16.}$$

And finally,

$$\underset{(\alpha = .10)}{\text{critical value}} = \frac{.90 - (.2995)}{1.9194} = .3129 \qquad (18)$$

This new critical value is more accurate (Ref 9) and it is probable that it will have a noticeable effect on the power study. It is also easily seen that this critical value will vary with statistics calculated from random samples. Therefore, the number of samples needed to obtain consistent results must be determined.

<u>Three Test Statistics</u>

As mentioned in Chapter I, the three test statistics that are used in this thesis are the Kolmogorov-Smirnov (K-S), the Anderson-Darling (A-D), and the Cramer-Von Mises test statistics. This section will briefly review and explain each of these statistics. Also, a short example is presented to illustrate each statistic. Although the research in this thesis was conducted for both the MLE and the BLUE, only the BLUE method is shown in the examples. It is noted that the examples for each of the statistics employ the same ten random numbers and thus the same values for the CDF.

Kolmogorov-Smirnov (K-S) Statistic. There are several tests that have been developed that relate to K-S criteria. Some of these are presented by Green and Hegazy (Ref 7:204) but the one used in this research is noted as follows:

$$D = \sup |F(x) - S_n(x)| \tag{19}$$

This statistic is defined by both Lilliefors (Ref 11) and Massey (Ref 13). "D" is a common notation for the K-S statistic, F(x) is the uniform CDF value of the given data point, and $S_n(x)$ is the sample cumulative step function. There are two values of $S_n(x)$ for each data point and they are calculated by i/n and (i-1)/n where i is the rank of the ith ordered statistic and n is the number of data points in the sample. The value of F(x) is found by using the BLUE and the value of each data point as shown in the following equation.

$$F(x) = (R_i - \hat{a}) / (\hat{b} - \hat{a}) \tag{20}$$

where

R_i is the value of the ith data point

$\hat{a}$ is the lower BLUE bound which is found by Eq 5

$\hat{b}$ is the upper BLUE bound which is found by Eq 6

Table 3 shows how the K-S statistic is calculated for a given data sample set of R_i where i = 1, 2, 3, ..., 10.

Table 3

K-S Statistic Calculation

i	$S_n(x)$		R_i	$F(x)$	$F(x)-S_n(x)$	
1	0	.1	.0189	.091	.091	.009
2	.1	.2	.1666	.238	.138	.038
3	.2	.3	.2717	.342	.142	.042
4	.3	.4	.3170	.387	.087	.013
5	.4	.5	.5478	.616	.216*	.116
6	.5	.6	.5883	.657	.157	.057
7	.6	.7	.6416	.710	.110	.010
8	.7	.8	.6984	.766	.066	.034
9	.8	.9	.8028	.870	.070	.030
10	.9	1.0	.8424	.909	.009	.091

$\hat{a} = -.0727$ $\hat{b} = .9339$ $D = \sup |F(x)-S_n(x)| = .216$

Anderson-Darling (A-D) Statistic. In this statistic, the first step is to order the sample set of observed data. This can be annotated by $x_1 \leq x_2 \leq \ldots \leq x_n$ where n is the number of observations. Now let A^2 be the value of the A-D statistic (this is a common notation). The equation for the A-D statistic (Ref 2:765) is

$$A^2 = -n - \frac{1}{n} \sum_{j=1}^{n} (2j-1) \ [\ln F(x_j)+\ln(1-F(x_{n-j+1}))] \tag{21}$$

where

$F(x_j)$ is the CDF value of the jth ordered data point

$F(x_{n-j+1})$ is the CDF value of the (n-j+1)th data point. Table 4 is an example of the A-D statistic calculation with $M = \ln F(x_j)$ and $N = \ln (1-F(x_{n-j+1}))$.

Table 4

Calculation of A-D Statistic

j	x_j	$F(x_j)$	$F(x_{n-j+1})$	M	N	(2j-1)(M+N)
1	.0189	.091	.909	-2.397	-2.397	- 4.794
2	.1666	.238	.870	-1.436	-2.038	-10.422
3	.2717	.342	.766	-1.073	-1.452	-12.625
4	.3170	.387	.710	- .949	-1.237	-15.302
5	.5478	.616	.657	- .485	-1.070	-13.995
6	.5883	.657	.616	- .420	- .957	-15.147
7	.6416	.710	.387	- .343	- .489	-10.816
8	.6984	.766	.342	- .267	- .419	-10.290
9	.8028	.870	.238	- .140	- .272	- 7.004
10	.8424	.909	.091	- .095	- .095	- 3.610
						Σ = -104.005

$$A^2 = -10 - \frac{1}{10}(-104.005) = .4005$$

Cramer-Von Mises (CVM) Statistic. This statistic is also described by Anderson and Darling (Ref 2:766). Again, let n equal the sample size and let the CDF value of the jth be annotated by u_j. Also, the observations must be ordered as before. The CVM statistic is given by:

$$W^2 = \frac{1}{12n} + \sum_{j=1}^{n} \left(u_j - \frac{(2j-1)}{2n}\right)^2 \tag{22}$$

Table 5 contains an example of the calculations for this statistic.

Table 5

Calculations for the CVM Statistic

j	x_j	u_j	$\frac{2j-1}{2n}$	$(u_j - \frac{(2j-1)}{2n})^2$
1	.0189	.091	.05	.0017
2	.1666	.238	.15	.0077
3	.2717	.342	.25	.0085
4	.3170	.387	.35	.0014
5	.5478	.616	.45	.0280
6	.5883	.657	.55	.0115
7	.6416	.710	.65	.0036
8	.6984	.766	.75	.0003
9	.8028	.870	.85	.0004
10	.8424	.909	.95	.0017
				Σ = .0648

$$W^2 = \frac{1}{12(10)} + (.0648) = .07313$$

III. Procedures

This chapter explains, step by step, the procedures followed in the research for this thesis. The calculations used for the generation of the critical value tables and in the power study involved extensive use of several computer programs, all of which were written in FORTRAN 77. These programs were deliberately kept relatively short and simple because of the large sample sizes and the large amounts of time it took to generate data by Monte Carlo simulation. The Control Data Systems CDC 6600 computer at Wright-Patterson AFB, Ohio was used to run these programs.

Standard Critical Value Calculations

A whole new set of critical value tables were calculated for the A-D, K-S, and CVM test statistics. This had to be done because previously published tables (Ref 7:207) did not employ the same techniques of estimating parameters nor did they use the bootstrap interpolation technique as discussed in Chapter II. The programs for the calculation of these tables were all structured in a similar manner. The basic steps are outlined as follows:

1. A large number of statistics are calculated for a given test and stored in a vector array.

2. This array is then ordered from smallest to largest.

3. A plotting position procedure is executed that assigns a plotting position to each of the statistics.

4. The bootstrap interpolation technique is then applied to obtain the desired critical value at the desired significance level.

Using these steps, tables of critical values with sample sizes running from 3 to 50 for the K-S, A-D, and CVM tests were obtained. Figure 2 is a flow chart that shows how these steps are implemented in computer programs to obtain the critical values.

When calculating the critical values for the separate tests, a separate program was written for each test. Since the programs were kept simple, it was necessary to manually build the tables. This was done by modifying the bootstrap interpolation section of the program for each significant level. There are five significance levels considered, so each program was modified and run five times.

Determination of Number of Samples. As noted earlier, the critical values vary with the statistics calculated from random samples. In order to get consistent results, large numbers of statistics for each value of n need to be calculated. The question is, how many are necessary? From the literature, a common number used is

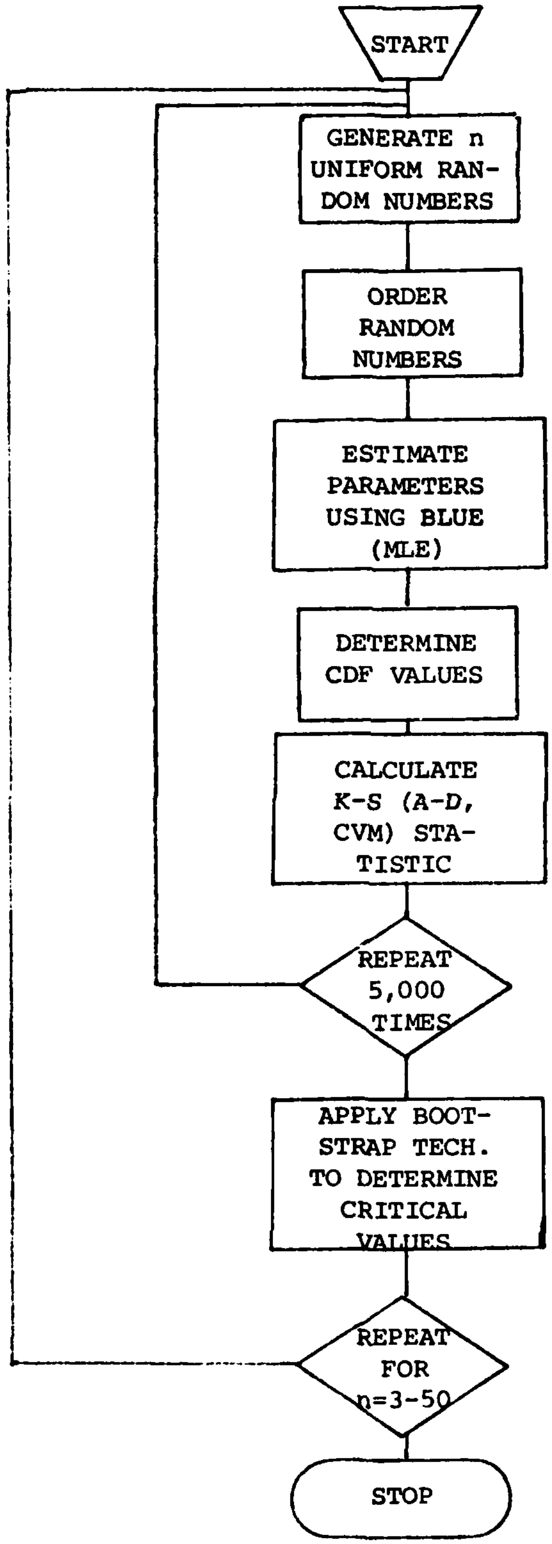

Figure 2. Flow Chart for Calculating Critical Values

5000. However, Green and Hegazy (Ref 7) used 10,000 as their sample size. Although this many would probably give more accurate critical values, it was found to be too burdensome for this system with its constraints. Because of the large amounts of time involved in running the programs and because of the constraints on computer availability, the testing of how many statistics it was necessary to calculate was kept to 5000 and below. In fact, the test for determining the best number was run using the K-S test statistic at sample sizes of 50, 100, 200, 500, 1000, and 5000. Different seeds were used for all trials.

<u>Plotting Position Procedure</u>. Since each of the statistics is matched with a plotting position, it is a simple matter to program the plotting position formula, given by Eq 10, directly after the procedure for statistics calculation.

Reflected Critical Value Calculations

After the standard tables for the three statistics were built, it was necessary to build similar tables for the same statistics using the reflection routine as described in Chapter II. This was accomplished by modifying the sections of the previous programs that generates the random numbers. A subroutine was added that did the reflecting of the data points about the mean. With this slight modification and the addition of the reflection subroutine, the programs were run again for the various test statistics at the various significance levels. The critical values obtained from these runs were then put into tables. Figure 3 is a flow chart of the reflection subroutine that is used in the programs.

Power Study

With tables of critical values for standard uniform data and for uniform data that are reflected about the sample mean, a comprehensive power study is now able to be accomplished. What is being done in the power study is that the powers of the standard (non-reflected) uniform critical values are being compared to those of the reflected critical values. It is hoped that the reflection procedure will give better or higher powers than the standard technique.

Recall that the null hypothesis in the test is that the sample data fit a known distribution (the uniform distribution in this case). The alternate hypothesis is that the sample data do not fit this distribution. Recall also that the power is the probability that the null hypothesis is rejected when the alternate hypothesis is true. These facts form the basis for the power study. To illustrate how the power study is accomplished, consider the following example for the K-S test statistic:

1. A random data sample is drawn from a distribution other than the uniform distribution.

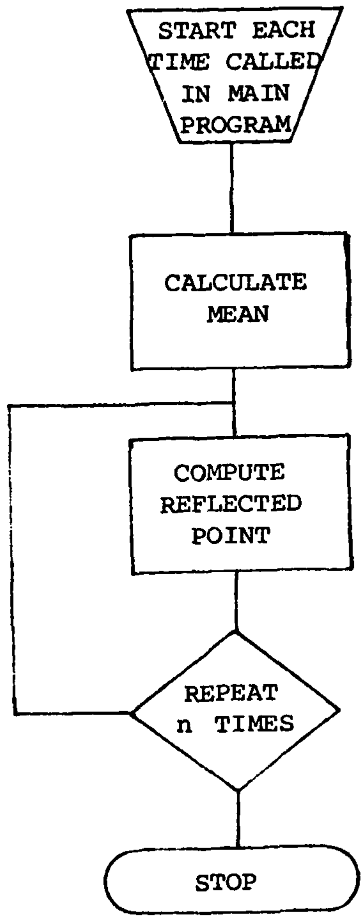

Figure 3. Flow Chart of Reflection Subroutine

2. The K-S test is run for this sample set to obtain the test statistic at a given significance level.

3. This statistic value is then compared to the appropriate table at the appropriate significance level.

4. If the statistic value is greater than or equal to the table value, the null hypothesis is rejected.

5. Steps 1-4 are accomplished 5000 times, each time using a different seed.

6. The number of times the null hypothesis is rejected is added up and divided by 5000. This gives the power value for that particular run.

Figure 4 is a flow chart that shows how these steps were implemented in computer programs for the power study. The above steps are accomplished for data sample sets of sizes 10, 20, 30, 40, and 50. This provided good comparisons for both large and small sample sizes. There are six statistics calculated for each of these sample sizes. These statistics are the K-S and the K-S reflected, the A-D and the A-D reflected, the CVM and the CVM reflected. A separate program is written for each of these six test statistics in the power study. The combining of programs and procedures was avoided in order to keep the calculations simple and to allow for manual construction of the power tables. The individual programs for the six statistics calculates the power of the given statistic at five different levels of significance (α = .01, .05, .10, .15, .20)

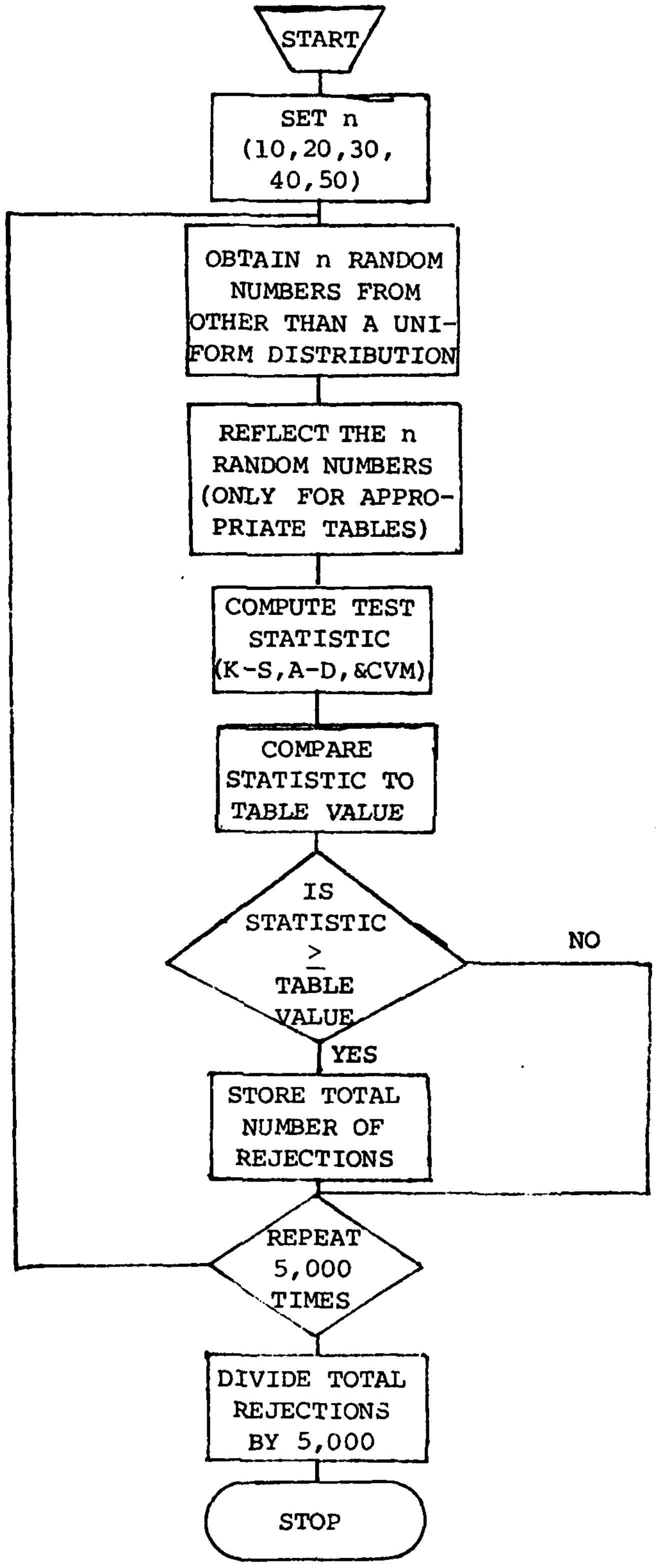

Figure 4. Flow Chart for Calculating Powers

and at a given value of n, the sample size. Since there are five sample sizes considered, each program is modified and run five times.

Distributions Used in Power Study. There are four distributions used in this power study. The study tests the uniform distribution versus the normal, Cauchy, triangular, and the double exponential distributions. The reason these distributions are presented is because of their symmetric properties. In his research concerning the normal distribution, Ream (Ref 15) found that a power study involving non-symmetric distributions was of little use. In fact, the powers for non-symmetric distributions decreased when the reflection technique was used. Good results could not be obtained with non-symmetric distributions. In an effort to verify this for the uniform distribution, a power study was run for the exponential and the lognormal distributions. As expected, the results were that the reflection technique did not help the power. Again, the power was found to decrease when the reflection technique was applied to these non-symmetrical distributions. Because of this, they are not included in this thesis.

The IMSL library contains several subroutines that were used in this thesis, particularly in the power study. The subroutines used to generate random numbers for the various distributions all came from the IMSL library. Also, a subroutine, that was particularly useful for ordering the various arrays from smallest to largest, was obtained from the IMSL library.

The only distribution considered that did not come from the IMSL library was the double exponential distribution. To generate random numbers for this distribution, continuous random numbers were first generated using an IMSL subroutine for generating uniform random numbers. The CDF for the double exponential is given below where the x_i's are the uniform random numbers.

$$F(y_i) = \begin{cases} \frac{1}{2} e^{yi} & \text{for } y_i \leq 0 \\ 1 - \frac{1}{2} e^{-y_i} & \text{for } y_i \geq 0 \end{cases} \tag{23}$$

$$\text{Now if } x_i \leq 0.5, \text{ then } y_i = \ln (2x_i) \text{ and if} \tag{24}$$

$$x_i > 0.5, \text{ then } y_i = - \ln (2-2x_i) \tag{25}$$

The y_i is then a pseudo-random sample from the double exponential distribution where i = 1, 2, 3, ..., n (Ref 12:265).

<u>MLE</u> <u>vs</u> <u>BLUE</u>

The procedures as mentioned in this chapter are first carried out using the MLE. All the tables of critical values are generated and the power studies for the above mentioned symmetric distributions are accomplished first with the MLE method for estimating the parameters, a and b. These same procedures are then reaccomplished using the BLUE method for estimating the parameters, a and b. This is done to determine what effect the changing of the method for estimating the parameters has on the power values.

Format of the Appendices

All of the tables that were generated for this research are listed in the appendices. The format of the appendices is as follows:

1. Appendices A-C contain all of the critical value tables (standard and reflected for both MLE's and BLUE's) for the K-S, A-D, and CVM statistics respectively.

2. Appendices D-F contain the power tables (uniform vs the above four symmetric distributions) for the K-S, A-D, and CVM statistics respectively.

IV. Discussion of Results

The main result of this study is the obtaining of the critical value tables and the power tables that are listed in the appendices. There are, however, several points that need to be explained. The purpose of this chapter is to not only list the findings of the research, but also to explain why the results came out as they did. An explanation of the use of the tables is also included.

Tables of Critical Values

The tables of critical values (rejection tables) listed in the appendices, are calculated using 5000 samples, an interpolating procedure (bootstrap technique), two different methods for estimating the parameters of the data set, and a reflection technique for doubling the size of the sample. It is necessary to generate these tables instead of using those that have already been generated by other authors (i.e., Green and Hegazy) because of these modifications that are used. It is because of these different procedures that there are differences in the tables. The fact that there are differences in the table values from different sources is not a critical issue. The important issue is that one must be consistent in the use of the tables. When running a test, the table that is used must be one that is generated in the same manner as the test. For example, if the tables that Green and Hegazy built are to be used, the test for goodness of fit must be based on the same criteria as the tables.

One interesting observation is that the critical values for the reflected K-S test are generally less than those for the standard, non-reflected K-S test. However, the critical values for the reflected CVM and A-D tests are generally larger than those for the standard CVM and A-D tests. As the sample sizes increase there are more exceptions that are observed.

The tables are set up for sample sizes running from 3 to 50 and for significance levels of 1, 5, 10, 15, and 20 percent (α = .01, .05, .10, .15, .20). By having the sample size run from 3 to 50, it is possible to obtain a power study that covers both large and small samples. It is also observed that as the sample size gets larger, the critical values look as if they are approaching some limit. The variation in the critical values becomes less as the sample size increases.

The use of the tables follows the standard hypothesis testing procedure when using the rejection or critical value tables. The steps involved in the use of these tables are outlined below:

1. Sample data are collected (for these tables, any sample size from 3 to 50 will do).

2. Double the sample size by employing the reflection technique as described in Chapter II. If a

standard, non-reflected test is being used, this step is eliminated and the tables that were generated without the reflection technique are referenced.

3. Calculate the desired statistic (K-S, A-D, CVM) as described in Chapter II.

4. Enter the rejection table of the desired statistic at the appropriate level of significance.

5. Compare the table value with the calculated statistic found in step 3.

6. If the calculated statistic is greater than or equal to the table value, reject the null hypothesis (H_0) that the sample data comes from a uniform distribution.

Power Study

As mentioned earlier, the power of a test is the probability that the null hypothesis will be rejected when the alternate hypothesis is true. The power study was accomplished with symmetric distributions only. Based on the studies done by others (Refs 15; 19) as well as two trials done by this author, the decision was made to eliminate non-symmetrical distributions. In all asymmetric cases, it was found that there was a decrease in power rather than an increase when the reflection procedure was applied.

It is observed from the power tables for MLE's that when the sample sizes are small, the reflection technique is generally not helpful in increasing the power.

When the sample size is increased to around 30, the reflection technique causes an increase in power. This is true with all distributions considered here with the exception of the Cauchy distribution. At all sample sizes, the reflection technique yields lower power values than the standard non-reflected technique. The distributions that show the best improvement in power with the reflection technique are the normal and the triangular distributions. The double exponential distribution generally is more sketchy. The power with this distribution is better at high sample sizes ($n = 30 - 50$) and at higher percentage significance levels ($\alpha = .01, .05$, and $.10$).

The power tables for the BLUE's show that higher powers can be obtained at all values of n when the reflection technique is applied in the normal and triangular cases. Those tables involving the Cauchy and double exponential distributions have similar results as the ones calculated with the MLE's.

Reading the power tables is basically self-explanatory. The power values corresponding to the single asterisk are for the standard non-reflected test. Those corresponding to the double asterisks are for the reflected tests.

MLE vs BLUE Comparison

The first major observation is concerned with the A-D statistic using the MLE's. The reader will observe that the power values when using MLE's behave quite strangely for the various distributions. This is not only with respect to the reflection-nonreflection comparison, but also with respect to the sample sizes. Consider the A-D power table for the triangular distribution. There is a dramatic increase in the power by applying the reflection technique and, although such an increase is possible, it would not be expected to be so great. When the power of the reflected technique for n = 10 is compared to that for n = 20, it is seen that the power decreases. Not only is this not expected, but it is also not logical. The reason behind these problems appears to be in the calculation of the A-D statistic. This statistic uses the natural log of the CDF in its calculation. When the MLE is used for estimating the parameters, finding the natural log of zero is a result. When the reflection technique is used, the finding of the natural log of zero occurs twice. Since the ln of zero is undefined, the computer programs for this statistic had to be altered so that whenever the natural log of zero came up, it was replaced by another number that was close to zero and that was arbitrarily set. This setting of an arbitrary number close to zero appears to cause the inconsistent results in the A-D power tables. When this first occurred, it was decided that a new method for estimating the parameters was needed. Thus, the BLUE technique as described in Chapter II is used. The results, as shown in Appendices D-F for the BLUE, give much better and consistent results.

Another result of using the BLUE's is that the powers are in general increased over those for the MLE's. The exceptions again are the Cauchy distribution and some of the lower sample sizes. This means that generally, when the BLUE's are used, the null hypothesis is rejected when the alternate hypothesis is true more often than when the MLE's are used. In other words, by using the BLUE's the possibility of Type II errors is less than if MLE's are used.

<u>Sample Size</u>

For the various K-S runs that were made with the sample sizes set at 50, 100, 200, 500, 1000 and 5000, the only one that gave consistent results at all alpha levels was 5000. It was observed that at 1000, consistent results were obtained for some of the significant levels but not for all. It wasn't until 5000 was tested that consistent results at all alpha levels are obtained. This is not to say that 5000 is the optimum number. Higher numbers could not be tested because of resource constraints and numbers between 1000 and 5000 were not tested because of the time constraint.

V. Conclusions and Recommendations

One of the principal objectives of this thesis is to verify an increase in the power of a goodness of fit test when a reflection technique is used. The concept of reflecting data about the sample mean comes from E. F. Schuster, who predicted that the use of this technique can be helpful when testing with symmetrical distributions (Ref 19). Schuster also stated that the statistic that is modified with this reflection technique would generally be better than the same statistic without the reflection technique (Ref 18). These assertions are being tested in this thesis for the uniform distribution using the three test statistics as outlined in Chapter II.

Conclusions

If an analyst can assume the data come from a symmetric distribution, the procedures of this thesis can be useful. The data can be put into a goodness of fit test as outlined in Chapter II. The procedures of Chapter III can be applied to test the hypothesis that the data correspond to a uniform distribution. If the test is accomplished and the null hypothesis is accepted, then the power study can be referenced.

From observation of the power tables it can be seen that the powers calculated for the given symmetric distributions versus the uniform distribution are generally better (more powerful) for the statistics employing the reflection procedure. The exceptions being those pertaining to the Cauchy distribution and a few of the lower sample sizes. The problem with the Cauchy distribution is not deemed very serious because of the very high power exhibited for both the standard, non-reflected procedure and the reflected procedure. Again, there is the problem of relatively low power shown for the smaller sample sizes.

It is also concluded that when you have the uniform versus non-symmetrical test, the reflection procedure is not helpful. In fact, it was found to yield lower powers when it was used with non-symmetrical distributions. This was predicted by Schuster (Ref 18), verified by Ream (Ref 15), and further substantiated by this author in trials run with the exponential and lognormal distributions.

Another important conclusion that is drawn from this research is that when testing the uniform distribution with unknown parameters, the best method of estimating those parameters is by using BLUE's. The reasons for this are: (1) they yield higher power values than the MLE's in all cases, (2) the problem of taking the natural log of zero in the A-D statistic is avoided, (3) the results of the power study for all three test statistics are more realistic and more useful, and (4) even with small sample sizes, the reflection technique when used with BLUE's increased the power for the normal and the triangular distributions in all three test statistics and it increased the power for all sample sizes for the double exponential distribution in the A-D test.

Although using BLUE's increases power, there still exists relatively low power values when n is small. For these goodness of fit tests, sample sizes greater than 20 are recommended. However, if this is not possible, it should still be remembered that the reflection technique still improves the power for small sample sizes.

Table 6 is used as a summary to list the statistics with the highest power at each of the alpha levels and sample sizes. For the reasons just noted, the table only includes those statistics calculated with the BLUE's. This table may be compared (Ref 15:64). The letters R and S in the parentheses signify whether the statistic is standard or reflected. As seen here, the predominately most powerful test is the A-D test. It is also observed that the powers for the CVM test are quite similar to the K-S test (Appendices D and F).

Recommendations

The critical values were generated for sample sizes running from 3 to 50. When the sample sizes got large (i.e., 40 to 50) the critical values seemed to be converging to some number. With larger sample sizes, further

Table 6

Highest Powered Statistics when Uniform Critical Values are Tested Using Four Symmetrical Alternative Distributions

Distribution Tested	n	Significance Level $\alpha=.01$	.05	.10	.15	.20
Normal	10	A-D(R)	A-D(R)	A-D(R)	A-D(R)	A-D(R)
	20	A-D(R)	A-D(R)	A-D(R)	A-D(R)	A-D(R)
	30	A-D(R)	A-D(R)	A-D(R)	A-D(R)	A-D(R)
	40	A-D(R)	A-D(R)	A-D(R)	A-D(R)	A-D(R)
	50	A-D(R)	A-D(R)	A-D(R)	A-D(R)	A-D(R)
Cauchy	10	K-S(S)	K-S(S)	K-S(S)	K-S(S)	K-S(S)
	20	K-S(S)	K-S(S)	A-D(S)	A-D(S)	A-D(S)
	30	K-S(S)	CVM(S)	A-D(S)	A-D(S)	A-D(S)
	40	A-D(S)	A-D(S)	A-D(S)	A-D(S)	A-D(S)
	50	K-S(S)	A-D(S)	A-D(S)	A-D(S)	A-D(S)
Double Exponential	10	K-S(S)	CVM(S)	A-D(R)	A-D(R)	A-D(R)
	20	K-S(S)	A-D(R)	A-D(R)	A-D(R)	A-D(R)
	30	A-D(R)	A-D(R)	A-D(R)	A-D(R)	A-D(R)
	40	A-D(R)	A-D(R)	A-D(R)	A-D(R)	A-D(S)
	50	K-S(R)	A-D(R)	A-D(S)	A-D(S)	A-D(S)
Triangular	10	CVM(R)	K-S(R)	K-S(R)	K-S(R)	K-S(R)
	20	CVM(R)	CVM(R)	CVM(R)	CVM(R)	CVM(R)
	30	CVM(R)	K-S(R)	K-S(R)	K-S(R)	K-S(R)
	40	K-S(R)	K-S(R)	K-S(R)	K-S(R)	K-S(R)
	50	K-S(R)	K-S(R)	CVM(R)	CVM(R)	CVM(R)

study might show a greater trend toward convergence, not only in critical values but also in the power values.

This thesis only looks at four distributions in the power study. Other distributions would make this study more comprehensive.

Another suggestion is that other kinds of test statistics could be used in a study of this nature. The K-S, A-D, and CVM test statistics are popular but it is possible that different tests might yield better results.

Finally, the method for estimating parameters obviously has an effect on the critical values and on the power values that are generated. Different methods of estimating parameters should be investigated to see if better, more accurate results can be obtained.

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Appendix A

Critical Value Tables of the K-S and the Modified K-S Statistics for the Uniform Distribution Using MLE and BLUE

Critical Values for the K-S Statistic
for the Uniform Distribution
(Parameters Estimated with the MLE)

	Alpha Level				
n	.01	.05	.10	.15	.20
3	.66198	.64207	.61770	.59230	.56678
4	.67323	.58734	.52329	.49275	.48086
5	.62659	.53992	.50039	.46618	.43299
6	.58798	.50672	.45993	.43275	.41055
7	.55672	.47179	.42564	.40016	.38099
8	.52278	.44479	.40267	.37614	.35586
9	.49236	.41846	.38199	.35571	.33643
10	.47285	.40103	.36459	.34019	.32029
11	.44566	.38335	.34516	.32203	.30598
12	.43256	.36957	.33327	.31126	.29447
13	.42541	.35536	.32374	.39951	.28286
14	.40528	.34235	.30999	.28870	.27406
15	.39923	.33287	.29904	.27788	.26303
16	.38512	.32221	.29005	.27112	.25526
17	.37882	.31471	.28329	.26357	.24845
18	.36545	.30419	.27781	.25854	.24222
19	.35860	.29842	.26779	.24978	.23597
20	.34227	.28956	.26153	.24248	.22998
21	.33769	.28400	.25563	.23895	.22492
22	.32620	.27828	.25134	.23333	.21877
23	.32899	.27277	.24639	.22802	.21503
24	.31895	.26735	.24120	.22388	.21016
25	.31042	.26216	.23539	.21919	.20706

n	Alpha Level .01	.05	.10	.15	.20
26	.30567	.25691	.23181	.21522	.20307
27	.29538	.24988	.22637	.21143	.19912
28	.29496	.24780	.22277	.20728	.19579
29	.28976	.24155	.21811	.20313	.19233
30	.28372	.23908	.21603	.20059	.18897
31	.28144	.23671	.21075	.19714	.18657
32	.27610	.23148	.20921	.19390	.18264
33	.26971	.22860	.20528	.19156	.18045
34	.27020	.22553	.20282	.18815	.17767
35	.26536	.22080	.19863	.18562	.17498
36	.25893	.21829	.19609	.18310	.17312
37	.25678	.21476	.19413	.18111	.17115
38	.25881	.21351	.19239	.17844	.16873
39	.24814	.21040	.18911	.17713	.16720
40	.25216	.20653	.18647	.17408	.16460
41	.24952	.20533	.18443	.17158	.16317
42	.24475	.20350	.18386	.17035	.16092
43	.24232	.20007	.18013	.16856	.15941
44	.23889	.19836	.17910	.16767	.15743
45	.28313	.19624	.17772	.16562	.15542
46	.23639	.19464	.17642	.16344	.15458
47	.23489	.19406	.17500	.16113	.15184
48	.23145	.19166	.17255	.16005	.15071
49	.23005	.18977	.16952	.15848	.14908
50	.22761	.18685	.16840	.15653	.14791

Critical Values of the Modified K-S Statistic
for the Uniform Distribution
(Parameters Estimated with the MLE)

	Alpha Level				
n	.01	.05	.10	.15	.20
3	.49531	.47534	.45103	.42564	.40012
4	.45242	.38895	.36561	.35382	.33965
5	.40372	.35271	.31874	.29777	.28848
6	.36390	.31242	.28630	.26705	.25242
7	.32804	.28253	.26056	.24581	.23270
8	.30415	.26302	.24185	.22732	.21468
9	.29025	.24531	.22376	.20875	.19861
10	.26920	.23012	.21126	.19664	.18613
11	.26135	.21796	.19929	.18496	.17583
12	.24405	.20849	.18811	.17627	.16778
13	.23608	.19944	.18056	.16821	.15978
14	.22521	.19069	.17252	.16167	.15303
15	.21650	.18301	.16684	.15535	.14757
16	.20921	.17807	.16173	.14949	.14060
17	.20447	.17356	.15671	.14485	.13688
18	.19839	.16805	.15146	.14101	.13217
19	.19336	.16189	.14654	.13629	.12807
20	.19024	.15876	.14286	.13305	.12476
21	.18656	.15561	.13960	.12964	.12200
22	.18353	.15064	.13582	.12613	.11868
23	.17657	.14826	.13246	.12319	.11650
24	.17281	.14394	.13020	.12032	.11385
25	.17015	.14157	.12668	.11740	.11048

	Alpha Level				
n	.01	.05	.10	.15	.20
26	.16664	.13674	.12419	.11465	.10829
27	.16403	.13398	.12093	.11250	.10609
28	.16026	.13176	.11813	.11010	.10428
29	.15495	.12970	.11654	.10793	.10181
30	.15168	.12754	.11368	.10582	.10030
31	.14904	.12601	.11151	.10396	.09866
32	.14700	.12204	.11010	.10239	.09689
33	.14551	.12087	.10840	.10056	.09475
34	.14141	.11811	.10685	.09932	.09381
35	.14116	.11632	.10443	.09805	.09290
36	.13868	.11506	.10355	.09625	.09092
37	.13550	.11408	.10241	.09500	.08945
38	.13429	.11245	.10114	.09383	.08854
39	.13016	.11052	.09952	.09280	.08721
40	.13097	.10868	.09814	.09131	.08606
41	.12933	.10762	.09664	.09003	.08547
42	.12809	.10681	.09542	.08907	.08399
43	.12503	.10507	.09413	.08802	.08309
44	.12345	.10378	.09378	.08717	.08252
45	.12158	.10194	.09282	.08654	.08151
46	.11961	.10102	.09149	.08591	.08056
47	.11773	.10020	.09031	.08429	.07952
48	.11597	.09938	.09005	.08301	.07824
49	.11534	.09850	.08865	.08248	.07788
50	.11532	.09694	.08766	.08167	.07665

Critical Values of the K-S Statistic
for the Uniform Distribution
(Parameters Estimated with the BLUE)

	Alpha Level				
n	.01	.05	.10	.15	.20
3	.86106	.69514	.60714	.55916	.51511
4	.73306	.59316	.52387	.47848	.45032
5	.65611	.52602	.47447	.44043	.41224
6	.59306	.49034	.44172	.40803	.38542
7	.53639	.45571	.40920	.37982	.35796
8	.51349	.42715	.38782	.35968	.33975
9	.48402	.40728	.36715	.34093	.32070
10	.46161	.38943	.35169	.32728	.30901
11	.43877	.37448	.33668	.31354	.29488
12	.43569	.35679	.32500	.30245	.28515
13	.41817	.34868	.31462	.29070	.27423
14	.40081	.33724	.30399	.28146	.26467
15	.39396	.32836	.29498	.27341	.25796
16	.38076	.31807	.28667	.26521	.24889
17	.37120	.31990	.27826	.25852	.24260
18	.35843	.30206	.27042	.25248	.23704
19	.35494	.29378	.26367	.24406	.22976
20	.34693	.28648	.25713	.23863	.22487
21	.33606	.27963	.25306	.23433	.22090
22	.33047	.27547	.24667	.22906	.21624
23	.32159	.26980	.24180	.22556	.21128
24	.31702	.26442	.23720	.21973	.20739
25	.30795	.25926	.23259	.21654	.20341

n	Alpha Level				
	.01	.05	.10	.15	.20
26	.30104	.25487	.22870	.21253	.19977
27	.29564	.24881	.22370	.20725	.19561
28	.29081	.24570	.21979	.20405	.19243
29	.28530	.23932	.21533	.20094	.18918
30	.28296	.23672	.21256	.19775	.18618
31	.27958	.23277	.20925	.19358	.18241
32	.27784	.22721	.20579	.19134	.18050
33	.26856	.22443	.20220	.18866	.17810
34	.26916	.22031	.19881	.18615	.17524
35	.26687	.21860	.19531	.18288	.17289
36	.26239	.21485	.19482	.18087	.17083
37	.25515	.21401	.19188	.17885	.16843
38	.25540	.20970	.18927	.17597	.16530
39	.25178	.20924	.18754	.17477	.16463
40	.25252	.20598	.18415	.17205	.16231
41	.24826	.20299	.18290	.17035	.16024
42	.24294	.20145	.18164	.16861	.15913
43	.23978	.19774	.17957	.16754	.15761
44	.24039	.19602	.17714	.16530	.15608
45	.23871	.19461	.17658	.16378	.15427
46	.23365	.19320	.17432	.16166	.15307
47	.23150	.19256	.17302	.16025	.15166
48	.22967	.18979	.17132	.15925	.14943
49	.22722	.18820	.16885	.15728	.14828
50	.22632	.18750	.16717	.15520	.14651

Critical Values of the Modified K-S Statistic
for the Uniform Distribution
(Parameters Estimated with the BLUE)

	Alpha Level				
n	.01	.05	.10	.15	.20
3	.88567	.74028	.64306	.57185	.50714
4	.76213	.60100	.50104	.43645	.39416
5	.65609	.49915	.41443	.36382	.33097
6	.58463	.43174	.35420	.31305	.28500
7	.49615	.36652	.31305	.28006	.25849
8	.46044	.33578	.28397	.25592	.23746
9	.42059	.30786	.26139	.23513	.21759
10	.38953	.28485	.24232	.21932	.20465
11	.36212	.26851	.22870	.20788	.19305
12	.33282	.24844	.21493	.19677	.18352
13	.31572	.23551	.20474	.18753	.17620
14	.29847	.22520	.19455	.18019	.16905
15	.28663	.21458	.18845	.17248	.16237
16	.26868	.20679	.18192	.16696	.15641
17	.25791	.19795	.17415	.16143	.15093
18	.24837	.19232	.16910	.15599	.14637
19	.24302	.18378	.16427	.15162	.14164
20	.23388	.18079	.15880	.14737	.13756
21	.22727	.17388	.15553	.14253	.13402
22	.21342	.16986	.15148	.13895	.13070
23	.20808	.16672	.14694	.13621	.12702
24	.20128	.16243	.14442	.13287	.12465
25	.19277	.15692	.14016	.12998	.12192

n	Alpha Level .01	.05	.10	.15	.20
26	.18677	.15521	.13756	.12687	.11929
27	.18467	.15054	.13427	.12425	.11631
28	.17871	.14686	.13155	.12215	.11394
29	.17586	.14401	.12899	.11910	.11198
30	.17282	.14137	.12080	.11640	.10888
31	.16927	.13841	.12334	.11404	.10728
32	.16768	.13466	.12152	.11224	.10522
33	.16227	.13255	.11934	.11032	.10338
34	.15875	.13049	.11648	.10786	.10184
35	.15619	.12876	.11442	.10580	.10015
36	.15434	.12649	.11240	.10458	.09836
37	.15187	.12430	.11136	.10315	.09691
38	.14888	.12330	.10941	.10087	.09526
39	.14652	.12001	.10756	.10002	.09432
40	.14376	.11796	.10608	.09860	.09279
41	.14229	.11699	.10442	.09711	.09192
42	.14111	.11427	.10392	.09629	.09095
43	.13869	.11345	.10176	.09511	.08974
44	.13471	.11192	.10124	.09406	.08936
45	.13426	.11072	.10022	.09330	.08811
46	.13105	.10887	.09856	.09200	.08690
47	.13106	.10809	.09735	.09075	.08547
48	.13067	.10653	.09647	.09000	.08499
49	.12747	.10502	.09533	.08899	.08398
50	.12634	.10424	.09378	.08823	.08306

Appendix B

Critical Value Tables of the A-D and the Modified A-D Statistics for the Uniform Distribution Using MLE and BLUE

Critical Values of the A-D Statistic
for the Uniform Distribution
(Parameters Estimated with the MLE)

n	Alpha Level .01	.05	.10	.15	.20
3	8.5430	6.8575	6.1926	5.8164	5.5543
4	7.3371	5.9839	5.3106	4.9321	4.6697
5	6.7747	5.4023	4.7127	4.3516	4.1077
6	6.4976	5.0555	4.3512	3.9769	3.6940
7	6.3342	4.6829	4.0782	3.6625	3.3995
8	5.9315	4.4861	3.8213	3.4633	3.1877
9	5.8105	4.2479	3.6272	3.2670	2.9618
10	5.4777	4.0536	3.4461	3.0885	2.8339
11	5.4312	3.9635	3.3077	2.9686	2.7081
12	5.3987	3.8083	3.1962	2.8489	2.6086
13	5.0864	3.6940	3.0863	2.7510	2.5081
14	5.0313	3.5643	2.9947	2.6847	2.4419
15	4.9199	3.5396	2.9218	2.6047	2.3965
16	4.7975	3.4307	2.9194	2.5416	2.3284
17	4.8121	3.3791	2.8330	2.5120	2.2694
18	4.7954	3.3365	2.7766	2.4394	2.2151
19	4.7898	3.3145	2.7593	2.4230	2.1723
20	4.6567	3.3176	2.7100	2.3769	2.1474
21	4.6083	3.2885	2.6921	2.3337	2.1145
22	4.7088	3.2728	2.6516	2.3420	2.1003
23	4.5390	3.1896	2.6090	2.2989	2.0667
24	4.6133	3.1937	2.6080	2.2717	2.0302
25	4.7042	3.1460	2.5699	2.2595	2.0120

	Alpha Level				
n	.01	.05	.10	.15	.20
26	4.6984	3.1278	2.5302	2.2170	2.0087
27	4.6868	3.0703	2.4995	2.1932	1.9733
28	4.5401	3.1191	2.4940	2.1822	1.9718
29	4.5623	3.1621	2.5209	2.1610	1.9443
30	4.4439	3.1426	2.4847	2.1661	1.9313
31	4.5562	3.1220	2.4776	2.1471	1.9149
32	4.4859	3.1195	2.4544	2.0941	1.8862
33	4.3650	3.1059	2.4213	2.0896	1.8777
34	4.4479	3.1077	2.4065	2.0800	1.8633
35	4.4850	3.0542	2.3930	2.0698	1.8483
36	4.4410	3.0412	2.3966	2.0715	1.8500
37	4.3768	3.0179	2.3869	2.0399	1.8364
38	4.4533	2.9906	2.3757	2.0301	1.8192
39	4.3380	2.9291	2.3710	2.0327	1.8136
40	4.3356	2.9091	2.3418	2.0405	1.7952
41	4.4206	2.9588	2.3477	2.0206	1.8057
42	4.4293	2.9192	2.3492	2.0156	1.7840
43	4.3216	2.9382	2.3396	2.0156	1.7872
44	4.3807	2.9260	2.3439	2.0231	1.7824
45	4.2964	2.9076	2.3170	1.9909	1.7615
46	4.1999	2.9260	2.2975	1.9805	1.7543
47	4.2008	2.8801	2.2995	1.9535	1.7316
48	4.1678	2.8872	2.2938	1.9300	1.7142
49	4.2483	2.9150	2.2814	1.9542	1.7090
50	4.2651	2.9342	2.2655	1.9233	1.6985

Critical Values of the Modified A-D Statistic
for the Uniform Distribution
(Parameters Estimated with the MLE)

	Alpha Level				
n	.01	.05	.10	.15	.20
3	15.2881	12.4927	11.4018	10.7923	10.3737
4	12.5984	10.3501	9.3703	8.7049	8.2847
5	11.2623	8.9382	7.9543	7.3735	6.9650
6	10.1240	7.9956	7.0615	6.5221	6.0718
7	9.2640	7.2442	6.3278	5.7950	5.4060
8	8.6801	6.6350	5.7610	5.2429	4.8678
9	8.1384	6.2608	5.2822	4.7667	4.4250
10	7.5136	5.7656	4.9630	4.4563	4.1126
11	7.2203	5.4520	4.6080	4.1273	3.8305
12	6.9082	5.1481	4.4325	3.9092	3.5933
13	6.5438	4.9596	4.1424	3.7182	3.4040
14	6.3794	4.7180	3.9544	3.5138	3.2083
15	6.1046	4.5048	3.8087	3.3921	3.0929
16	5.9580	4.3950	3.6764	3.2602	2.9721
17	5.6821	4.2264	3.5609	3.1161	2.8364
18	5.6183	4.1225	3.4299	3.0152	3.7303
19	5.4793	3.9808	3.3348	2.9258	2.6433
20	5.4005	3.8053	3.2527	2.8426	2.5890
21	5.3547	3.8092	3.1523	2.7719	2.5012
22	5.1882	3.7652	3.0863	2.6856	2.4273
23	4.9936	3.6455	2.9775	2.6205	2.3464
24	4.9257	3.5900	2.8900	2.5360	2.2802
25	4.9180	3.4951	2.8335	2.4513	2.2085

	Alpha Level				
n	.01	.05	.10	.15	.20
26	4.8047	3.4007	2.8030	2.4074	2.1809
27	4.7765	3.3780	2.7570	2.3605	2.1201
28	4.7260	3.3259	2.7161	2.3269	2.0762
29	4.8720	3.2853	2.6565	2.2892	2.0518
30	4.6733	3.2044	2.6130	2.2497	2.0100
31	4.5721	3.1208	2.5566	2.2304	1.9804
32	4.4969	3.0899	2.5020	2.2132	1.9637
33	4.3720	3.0367	2.4831	2.1642	1.9415
34	4.3580	3.0479	2.4485	2.1355	1.9111
35	4.3458	2.9677	2.4354	2.0977	1.8728
36	4.2555	2.9574	2.4277	2.0686	1.8310
37	4.2840	2.9126	2.3701	2.0353	1.8158
38	4.1940	2.8585	2.3303	2.0082	1.7941
39	4.1662	2.8304	2.3069	1.9945	1.7607
40	4.1364	2.8084	2.2791	1.9623	1.7461
41	4.1145	2.7777	2.2468	1.9218	1.7124
42	4.0160	2.7458	2.2397	1.8981	1.6884
43	3.9760	2.7316	2.2002	1.8781	1.6701
44	3.9077	2.7199	2.1824	1.8435	1.6544
45	4.0014	2.6860	2.1378	1.8350	1.6275
46	3.9650	2.7080	2.1346	1.8285	1.6129
47	3.9581	2.6737	2.1486	1.8238	1.5977
48	3.8034	2.6551	2.1592	1.8151	1.5856
49	3.8431	2.6318	2.1201	1.7963	1.5733
50	3.8210	2.6305	2.1136	1.7793	1.5717

Critical Values of the A-D Statistic
for the Uniform Distribution
(Parameters Estimated with the BLUE)

	Alpha Level				
n	.01	.05	.10	.15	.20
3	4.0687	2.1752	1.5797	1.2654	1.0706
4	3.5021	1.9785	1.5001	1.2119	1.0381
5	3.2911	1.9424	1.4847	1.2197	1.0463
6	3.2897	2.0165	1.5290	1.2509	1.0732
7	3.1820	2.0698	1.5651	1.2834	1.0968
8	3.1401	2.0678	1.5860	1.2945	1.1170
9	3.2273	2.1136	1.5716	1.3302	1.1394
10	3.3750	2.0454	1.6290	1.3351	1.1169
11	3.2613	2.0685	1.6135	1.3208	1.1458
12	3.3956	2.1000	1.5827	1.3194	1.1490
13	3.2260	2.1117	1.6095	1.3314	1.1568
14	3.3943	2.0702	1.6044	1.3450	1.1697
15	3.3791	2.1355	1.6430	1.3936	1.2063
16	3.2946	2.1422	1.6544	1.3765	1.2026
17	3.4620	2.1653	1.6581	1.3939	1.2022
18	3.4753	2.1974	1.6747	1.4270	1.2361
19	3.4847	2.1689	1.7140	1.4250	1.2373
20	3.5162	2.2121	1.7102	1.4468	1.2416
21	3.5236	2.2255	1.7265	1.4578	1.2621
22	3.5284	2.2822	1.7523	1.4524	1.2562
23	3.6108	2.2454	1.7499	1.4560	1.2539
24	3.6389	2.2629	1.7541	1.4575	1.2488
25	3.7464	2.2457	1.7720	1.4645	1.2485

	Alpha Level				
n	.01	.05	.10	.15	.20
26	3.7679	2.3248	1.7571	1.4530	1.2574
27	3.6860	2.2894	1.7501	1.4697	1.2571
28	3.6012	2.3021	1.8009	1.4681	1.2699
29	3.7083	2.3761	1.7833	1.4761	1.2787
30	3.6288	2.3793	1.7857	1.4925	1.2899
31	3.5850	2.3545	1.7927	1.5049	1.2977
32	3.6640	2.3670	1.8281	1.4938	1.2883
33	3.5810	2.3785	1.7985	1.5042	1.2993
34	3.5743	2.3601	1.8271	1.4988	1.3103
35	3.5450	2.3409	1.8240	1.4887	1.2885
36	3.5388	2.3803	1.8238	1.4969	1.3048
37	3.6298	2.3693	1.7996	1.5137	1.3129
38	3.6124	2.3359	1.8097	1.5296	1.3117
39	3.6117	2.3170	1.8054	1.5191	1.3103
40	3.6181	2.3603	1.8070	1.5081	1.3085
41	3.6757	2.3811	1.8417	1.5059	1.3017
42	3.6711	2.3664	1.8238	1.5143	1.2978
43	3.6959	2.3450	1.8149	1.5467	1.3191
44	3.6292	2.3466	1.8478	1.5420	1.3099
45	3.5880	2.3564	1.8477	1.5153	1.3067
46	3.5493	2.3404	1.8260	1.5317	1.3119
47	3.5818	2.3101	1.8284	1.5125	1.3061
48	3.6322	2.3692	1.8183	1.5154	1.3078
49	3.5813	2.3799	1.8252	1.5121	1.2994
50	3.7611	2.4270	1.8253	1.4877	1.3074

Critical Values of the Modified A-D Statistic
for the Uniform Distribution
(Parameters Estimated with the BLUE)

	Alpha Level				
n	.01	.05	.10	.15	.20
3	9.5697	5.0756	3.6269	2.8201	2.2507
4	7.2814	4.1593	2.8658	2.2744	1.8814
5	6.6664	3.5181	2.5608	2.0120	1.6730
6	6.1571	3.2152	2.2695	1.8618	1.5736
7	5.4943	2.9150	2.1570	1.7610	1.5160
8	4.9806	2.7991	2.0625	1.6977	1.4208
9	4.8035	2.7131	1.9993	1.6076	1.3714
10	4.6578	2.5677	1.8991	1.5367	1.3152
11	4.5058	2.4895	1.8383	1.4955	1.2744
12	4.2729	2.4339	1.8089	1.4762	1.2671
13	4.0504	2.3544	1.7305	1.4431	1.2367
14	3.9124	2.2909	1.7008	1.4075	1.2104
15	4.1464	2.2415	1.6854	1.4044	1.2000
16	3.8606	2.1830	1.6450	1.3865	1.1833
17	3.8206	2.1636	1.6350	1.3734	1.1751
18	3.6881	2.1528	1.6214	1.3522	1.1569
19	3.4686	2.0737	1.5728	1.3249	1.1379
20	3.3136	2.0164	1.6030	1.3128	1.1248
21	3.3501	2.0391	1.5621	1.3204	1.1390
22	3.3544	2.0544	1.5748	1.3144	1.1343
23	3.1554	2.0002	1.5605	1.3164	1.1179
24	3.2034	1.9998	1.5306	1.2834	1.1332
25	3.1224	1.9335	1.5249	1.2768	1.0961

n	Alpha Level .01	.05	.10	.15	.20
26	3.1140	1.9839	1.5271	1.2603	1.0909
27	3.1299	1.9562	1.5046	1.2477	1.0806
28	3.0599	1.9576	1.5026	1.2440	1.0899
29	3.1557	1.9444	1.4974	1.2502	1.0690
30	3.0662	1.9201	1.4829	1.2370	1.0613
31	3.1405	1.9158	1.4728	1.2271	1.0683
32	2.9628	1.9145	1.4739	1.2350	1.0574
33	2.9090	1.8916	1.4658	1.2189	1.0567
34	2.9042	1.8942	1.4578	1.2029	1.0445
35	2.8856	1.8526	1.4512	1.1913	1.0517
36	2.8810	1.8560	1.4372	1.1939	1.0535
37	2.9157	1.8544	1.4308	1.1995	1.0465
38	2.8608	1.8423	1.4237	1.1814	1.0323
39	2.9815	1.8581	1.4152	1.1818	1.0323
40	2.9265	1.8209	1.4117	1.1908	1.0297
41	2.8851	1.8248	1.3979	1.1734	1.0224
42	2.2970	1.8152	1.3884	1.1729	1.0202
43	2.8688	1.8209	1.3960	1.1564	1.0195
44	2.7832	1.7963	1.3916	1.1659	1.0148
45	2.9015	1.8068	1.3894	1.1681	1.0073
46	2.8440	1.8269	1.3772	1.1800	1.0228
47	2.8452	1.8049	1.3728	1.1689	1.0191
48	2.8465	1.7865	1.3796	1.1783	1.0181
49	2.8330	1.7662	1.3760	1.1655	1.0116
50	2.8362	1.7583	1.3893	1.1672	1.0004

Appendix C

Critical Value Tables of the CVM and the Modified CVM Statistics for the Uniform Distribution Using MLE and BLUE

Critical Values of the CVM Statistic
for the Uniform Distribution
(Parameters Estimated with the MLE)

	Alpha Level				
n	.01	.05	.10	.15	.20
3	.32803	.30935	.28456	.26371	.24483
4	.47896	.36118	.29797	.25772	.23171
5	.55036	.37779	.31192	.26967	.23853
6	.57796	.39830	.31313	.26784	.23447
7	.59344	.39883	.31932	.27279	.23847
8	.60346	.40288	.32088	.27076	.23745
9	.60783	.41201	.32456	.26949	.23615
10	.62349	.40748	.31934	.26771	.23464
11	.66330	.42532	.32273	.27039	.23645
12	.65978	.42426	.33312	.27532	.24049
13	.65903	.42556	.32928	.27722	.24061
14	.67366	.44326	.34076	.27540	.23962
15	.68341	.43440	.33307	.27525	.23893
16	.69014	.43858	.33413	.27681	.23674
17	.68998	.43649	.33003	.27538	.23534
18	.69438	.43737	.33348	.28099	.23651
19	.69534	.43559	.33691	.27867	.23900
20	.68524	.43352	.33409	.27746	.24136
21	.68251	.43917	.33154	.27770	.23871
22	.68887	.43599	.32937	.27810	.23997
23	.68668	.44011	.33592	.27894	.23781
24	.68143	.43913	.33223	.28026	.24000
25	.69229	.43285	.33604	.27855	.23949

	Alpha Level				
n	.01	.05	.10	.15	.20
26	.72948	.43938	.33492	.27908	.23988
27	.71071	.44535	.33846	.28182	.23935
28	.70017	.43825	.33580	.28118	.23883
29	.70807	.44273	.33404	.27813	.23900
30	.72837	.45084	.34486	.28263	.23848
31	.72818	.46161	.34189	.28345	.24092
32	.72193	.45548	.34291	.28232	.24146
33	.71909	.45142	.34502	.28383	.24062
34	.70685	.46097	.34366	.28443	.24000
35	.72984	.46112	.34175	.28226	.24313
36	.71835	.45543	.33913	.28447	.23987
37	.68936	.45282	.34661	.28270	.24393
38	.69719	.45166	.34789	.28300	.24163
39	.69984	.45560	.34930	.28343	.24087
40	.70307	.45332	.34529	.28036	.23986
41	.71622	.45210	.34617	.27912	.24003
42	.72008	.45186	.34164	.28212	.24032
43	.71636	.45226	.34206	.27948	.24096
44	.70420	.45612	.34174	.28059	.24172
45	.70470	.45982	.34320	.28019	.23850
46	.69969	.45810	.34262	.28326	.24108
47	.72270	.45550	.34654	.28288	.24074
48	.70394	.46135	.33957	.27994	.24126
49	.72186	.46348	.34227	.28111	.24069
50	.72565	.45705	.34529	.28068	.24009

Critical Values of the Modified CVM Statistic
for the Uniform Distribution
(Parameters Estimated with the MLE)

	Alpha Level				
n	.01	.05	.10	.15	.20
3	.49117	.46022	.41960	.38590	.35570
4	.55244	.44058	.37036	.32014	.29136
5	.55637	.41572	.34599	.29958	.26500
6	.54868	.39143	.32273	.27811	.24244
7	.53486	.37724	.30617	.26138	.22530
8	.52805	.36465	.28828	.24826	.21542
9	.51115	.34564	.27815	.23401	.20370
10	.49088	.34049	.26994	.22616	.19421
11	.47641	.33113	.26316	.22278	.19037
12	.48118	.32757	.25719	.21334	.18370
13	.46628	.31420	.25218	.21018	.17887
14	.47912	.32109	.24597	.20230	.17546
15	.46276	.31070	.24451	.19868	.16983
16	.47636	.29914	.23440	.19459	.16598
17	.46751	.30567	.23170	.18863	.16345
18	.46704	.30161	.22679	.18635	.15946
19	.45342	.29712	.22682	.18775	.15868
20	.44435	.29664	.22497	.18437	.15671
21	.44903	.28945	.22413	.18394	.15543
22	.45874	.28936	.21978	.18104	.15322
23	.44392	.28368	.21456	.17893	.15168
24	.44631	.28461	.21563	.17967	.15054
25	.44185	.28298	.21240	.17524	.15076

	Alpha Level				
n	.01	.05	.10	.15	.20
26	.44630	.27870	.21381	.17374	.14850
27	.45579	.27304	.21243	.17422	.14781
28	.44546	.27632	.21271	.17113	.14567
29	.43870	.28373	.20751	.16979	.14392
30	.44351	.28059	.20628	.16822	.14521
31	.45389	.27878	.20339	.16981	.14202
32	.45091	.27491	.20707	.16950	.14085
33	.44784	.27614	.20666	.16791	.14247
34	.43878	.27602	.20355	.16756	.14164
35	.43934	.27162	.20283	.16630	.14275
36	.43306	.27509	.20238	.16908	.14248
37	.44050	.26934	.20360	.16655	.14098
38	.43904	.26866	.20308	.16641	.14019
39	.43230	.26734	.20630	.16478	.14012
40	.43495	.26488	.20249	.16366	.13912
41	.42764	.26510	.19916	.16431	.13901
42	.43187	.26489	.19958	.16216	.13949
43	.43568	.26646	.19621	.16249	.13765
44	.43090	.25990	.19782	.16301	.13603
45	.43374	.26522	.19797	.16184	.13557
46	.41933	.26458	.19373	.15931	.13846
47	.42206	.26786	.19578	.15679	.13117
48	.42903	.25937	.19396	.15655	.13218
49	.43136	.25986	.19424	.15751	.13226
50	.43289	.25542	.19346	.15549	.13311

Critical Values of the CVM Statistic
for the Uniform Distribution
(Parameters Estimated with the BLUE)

	Alpha Level				
n	.01	.05	.10	.15	.20
3	.68503	.41511	.28059	.22641	.18862
4	.64853	.37250	.27087	.21406	.17883
5	.61375	.36763	.27051	.22166	.18801
6	.60358	.36301	.27285	.22034	.18790
7	.58137	.36976	.27705	.22627	.18962
8	.60108	.36494	.27605	.22891	.19547
9	.61956	.36766	.28475	.23192	.20070
10	.62161	.37833	.28473	.23596	.20179
11	.61677	.37958	.29136	.23844	.20343
12	.62449	.39518	.29702	.24406	.20580
13	.62914	.39686	.30456	.24511	.20946
14	.62653	.40960	.30567	.24467	.20936
15	.64220	.40336	.30364	.25140	.21747
16	.63219	.40996	.30427	.24971	.21286
17	.68133	.40684	.30566	.24898	.21306
18	.66835	.40883	.31359	.25503	.21390
19	.66224	.40823	.30621	.25269	.21424
20	.65898	.41420	.30799	.25340	.21517
21	.67188	.42137	.31343	.25491	.21766
22	.65862	.41752	.31928	.25904	.21909
23	.67544	.42093	.31494	.26275	.22580
24	.68575	.41512	.31372	.26347	.22229
25	.68389	.41690	.31703	.26220	.22285

n	Alpha Level .01	.05	.10	.15	.20
26	.66182	.41131	.31876	.26547	.22543
27	.66791	.41875	.31995	.26351	.22778
28	.68322	.41434	.31911	.26600	.22464
29	.68293	.42232	.32272	.26198	.22236
30	.69932	.42401	.32577	.26312	.22435
31	.66620	.43345	.33113	.26482	.22641
32	.68664	.43814	.31951	.26644	.22784
33	.68450	.43905	.32267	.26776	.22909
34	.69483	.43892	.32303	.26986	.22959
35	.70099	.44047	.32468	.26882	.22790
36	.70283	.43471	.32540	.26449	.22959
37	.70075	.42776	.32681	.27118	.22669
38	.67773	.43374	.33358	.26964	.22627
39	.69045	.43415	.33050	.26997	.22850
40	.70860	.43947	.32830	.26932	.22745
41	.70674	.43959	.33259	.26640	.22994
42	.71473	.44096	.32987	.27192	.23135
43	.70693	.43988	.33107	.27296	.23124
44	.69235	.44275	.33123	.27466	.23255
45	.70076	.43317	.33690	.27188	.23113
46	.69561	.43634	.33543	.27306	.23017
47	.69685	.43472	.33359	.27596	.23279
48	.72336	.44138	.33729	.27563	.23318
49	.68891	.44642	.33353	.27496	.23395
50	.69810	.43915	.33647	.27544	.23175

Critical Values for the Modified CVM Statistic
for the Uniform Distribution
(Parameters Estimated with the BLUE)

n	Alpha Level .01	.05	.10	.15	.20
3	1.5086	1.0078	.69228	.52325	.39531
4	1.4099	.84207	.52675	.37882	.29366
5	1.3074	.67025	.43065	.32132	.25135
6	1.1814	.59698	.37628	.28016	.22771
7	1.1417	.52270	.34107	.26014	.21280
8	.99500	.48746	.31729	.24407	.20097
9	.86687	.43467	.30400	.23495	.19266
10	.84117	.41600	.29440	.22028	.18358
11	.76688	.40283	.27460	.21525	.17900
12	.70410	.38526	.26374	.21026	.17271
13	.68128	.36436	.25927	.20587	.17216
14	.64336	.34370	.25150	.20177	.16947
15	.62532	.32716	.24472	.19997	.16721
16	.61438	.32030	.23423	.19182	.16121
17	.62443	.30801	.23024	.18787	.16022
18	.55980	.31141	.22697	.18431	.15677
19	.54652	.29882	.22392	.18589	.15963
20	.53625	.30450	.22335	.18559	.15581
21	.53381	.29449	.22119	.18243	.15685
22	.51465	.29149	.22152	.18112	.15381
23	.49979	.28909	.21815	.17849	.15228
24	.49585	.28208	.21684	.17729	.15218
25	.46569	.28093	.21699	.17740	.15251

	Alpha Level				
n	.01	.05	.10	.15	.20
26	.46242	.27536	.21540	.17426	.15017
27	.45544	.28044	.21262	.17682	.14900
28	.44810	.27694	.21438	.17458	.14644
29	.45688	.28091	.21597	.17447	.14577
30	.44197	.28652	.21232	.17088	.14500
31	.42396	.28282	.21086	.16972	.14406
32	.42356	.28002	.20798	.17024	.14641
33	.44515	.28013	.20710	.17077	.14356
34	.44342	.27389	.20626	.17099	.14446
35	.43420	.27817	.21634	.16960	.14508
36	.42421	.27600	.20802	.16769	.14311
37	.43299	.27077	.20565	.16873	.14402
38	.41798	.27270	.20419	.16708	.14266
39	.43174	.27334	.20425	.16675	.14376
40	.44416	.27364	.20219	.16904	.14319
41	.44020	.26606	.20162	.16610	.14332
42	.43486	.26088	.20166	.16623	.13973
43	.41844	.26073	.19944	.16598	.13872
44	.42421	.25876	.19560	.16271	.14060
45	.42712	.25861	.19769	.16237	.13884
46	.43222	.25871	.19388	.16245	.13693
47	.42739	.26418	.19735	.16120	.13691
48	.42876	.25596	.19924	.16030	.13758
49	.42666	.25943	.19255	.16035	.13609
50	.43203	.25645	.19465	.15826	.13449

Appendix D

Power Tables for the K-S Statistic

Powers for Testing H_0: Population is Uniform,
When Actual Population is Normal
Kolmogorov-Smirnov Statistic

Maximum Likelihood Estimators

n	Calculation Method *- standard **- reflected	Powers at Alpha Level .01	.05	.10	.15	.20
10	*	.0176	.0712	.1310	.1820	.2328
10	**	.0034	.0170	.0378	.0678	.1014
20	*	.0636	.1786	.2736	.3514	.4136
20	**	.0280	.0996	.1668	.2268	.2844
30	*	.1482	.3216	.4448	.5268	.6010
30	**	.1720	.3478	.4692	.5408	.5928
40	*	.2446	.4946	.6920	.7104	.7648
40	**	.3938	.6250	.7222	.7792	.8190
50	*	.3534	.6390	.7564	.8236	.8656
50	**	.6340	.8052	.8666	.9020	.9292

Best Linear Unbiased Estimators

n	Calculation Method *- standard **- reflected	Powers at Alpha Level .01	.05	.10	.15	.20
10	*	.1300	.2560	.3494	.4244	.4764
10	**	.2338	.4158	.5292	.6082	.6604
20	*	.2046	.4186	.5304	.6112	.6704
20	**	.3678	.5988	.7066	.7578	.7968
30	*	.3552	.5418	.6534	.7338	.7892
30	**	.6144	.7836	.8834	.8978	.9284
40	*	.4314	.6560	.7672	.8302	.8724
40	**	.7854	.9134	.9472	.9640	.9734
50	*	.5126	.7200	.8300	.8874	.9214
50	**	.8816	.9542	.9792	.9846	.9896

Powers for Testing H_0: Population is Uniform,
When Actual Population is Cauchy
Kolmogorov-Smirnov Statistic

Maximum Likelihood Estimators

n	Calculation Method *- standard **- reflected	Powers at Alpha Level .01	.05	.10	.15	.20
10	*	.4878	.6066	.6722	.7172	.7552
10	**	.2746	.3660	.4196	.4608	.5044
20	*	.8446	.9144	.9434	.9614	.9682
20	**	.6184	.7490	.8876	.8424	.8764
30	*	.9592	.9852	.9930	.9958	.9978
30	**	.8566	.9252	.9518	.9628	.9708
40	*	.9920	.9988	.9992	.9998	1.0000
40	**	.9544	.9846	.9920	.9952	.9974
50	*	.9992	1.0000	1.0000	1.0000	1.0000
50	**	.9866	.9952	.9980	.9994	.9996

Best Linear Unbiased Estimators

n	Calculation Method *- standard **- reflected	Powers at Alpha Level .01	.05	.10	.15	.20
10	*	.4748	.6210	.6948	.7464	.7842
10	**	.0640	.2668	.4020	.4974	.5734
20	*	.8400	.9246	.9532	.9696	.9780
20	**	.4914	.7298	.8224	.8670	.9026
30	*	.9624	.9874	.9950	.9966	.9978
30	**	.8292	.9284	.9654	.9740	.9830
40	*	.9924	.9986	.9994	.9998	.9998
40	**	.9532	.9874	.9954	.9980	.9983
50	*	.9996	1.0000	1.0000	1.0000	1.0000
50	**	.9884	.9972	.9996	.9998	.9998

Powers for Testing H_o: Population is Uniform,
When Actual Population is Triangular
Kolmogorov-Smirnov Statistic

Maximum Likelihood Estimators

n	Calculation Method *- standard **- reflected	Powers at Alpha Level .01	.05	.10	.15	.20
10	*	.0104	.0420	.0876	.1342	.1804
10	**	.0034	.0172	.0346	.0634	.0966
20	*	.0156	.0702	.1316	.1966	.2442
20	**	.0096	.0506	.1004	.1444	.1960
30	*	.0260	.1074	.1900	.2728	.3504
30	**	.0656	.1864	.2966	.3682	.4286
40	*	.0364	.1714	.2970	.3912	.4722
40	**	.1644	.3834	.5044	.5848	.6480
50	*	.0554	.2292	.3954	.5044	.5884
50	**	.3280	.5654	.6804	.7482	.7978

Best Linear Unbiased Estimators

n	Calculation Method *- standard **- reflected	Powers at Alpha Level .01	.05	.10	.15	.20
10	*	.0196	.1034	.2078	.3024	.3710
10	**	.1032	.3806	.5304	.6264	.6840
20	*	.0436	.1990	.3450	.4674	.5630
20	**	.3230	.6312	.7544	.8120	.8570
30	*	.0942	.3322	.5060	.6166	.7038
30	**	.6090	.8140	.9114	.9250	.9440
40	*	.1336	.4502	.6504	.7446	.8150
40	**	.7860	.9228	.9612	.9724	.9820
50	*	.2016	.5390	.7386	.8358	.8874
50	**	.8894	.9656	.9828	.9894	.9940

Powers for Testing H_0: Population is Uniform,
When Actual Population is Double Exponential
Kolmogorov-Smirnov Statistic

Maximum Likelihood Estimators						
	Calculation Method	Powers at Alpha Level				
n	*- standard **- reflected	.01	.05	.10	.15	.20
10	*	.0894	.2120	.2886	.3508	.4110
10	**	.0068	.0326	.0622	.0984	.1332
20	*	.3122	.4930	.6004	.6832	.7298
20	**	.1308	.2844	.3820	.4532	.5162
30	*	.5250	.7298	.8210	.8750	.9094
30	**	.4588	.6316	.7174	.7648	.7972
40	*	.7012	.8858	.9426	.9632	.9778
40	**	.7124	.8422	.8898	.9148	.9312
50	*	.8298	.9520	.9810	.9924	.9962
50	**	.8694	.9312	.9562	.9726	.9806

Best Linear Unbiased Estimators						
	Calculation Method	Powers at Alpha Level				
n	*- standard **- reflected	.01	.05	.10	.15	.20
10	*	.1464	.2928	.3998	.4724	.5294
10	**	.1288	.2476	.3728	.4690	.5348
20	*	.3546	.5646	.6852	.7598	.8114
20	**	.2244	.5454	.6826	.7522	.8070
30	*	.5628	.7644	.8574	.9058	.9396
30	**	.6178	.8078	.8966	.9124	.9330
40	*	.7178	.9012	.9548	.9730	.9830
40	**	.8332	.9294	.9562	.9700	.9796
50	*	.8402	.9574	.9840	.9922	.9956
50	**	.9296	.9748	.9872	.9912	.9946

Appendix E

Power Tables for the A-D Statistic

Powers for Testing H_0: Population is Uniform,
When Actual Population is Normal
Anderson-Darling Statistic

Maximum Likelihood Estimators						
	Calculation Method *- standard	Powers at Alpha Level				
n	**- reflected	.01	.05	.10	.15	.20
10	*	.0140	.0500	.0898	.1280	.1626
10	**	.1610	.3574	.3638	.3714	.3828
20	*	.0336	.1028	.1756	.2356	.2872
20	**	.0480	.1842	.2714	.3608	.3996
30	*	.0662	.1828	.3322	.4358	.5200
30	**	.0804	.3186	.4602	.5636	.6412
40	*	.1324	.3946	.5646	.6732	.7474
40	**	.2850	.5880	.7176	.7920	.8386
50	*	.2674	.5836	.7610	.8392	.8892
50	**	.5584	.7890	.8772	.9228	.9446
Best Linear Unbiased Estimators						
	Calculation Method *- standard	Powers at Alpha Level				
n	**- reflected	.01	.05	.10	.15	.20
10	*	.0932	.1660	.2128	.2750	.3536
10	**	.2602	.4940	.6118	.7274	.7938
20	*	.2520	.3606	.4482	.5294	.6022
20	**	.5014	.7210	.8078	.8646	.8968
30	*	.3076	.4500	.5906	.6770	.7472
30	**	.6598	.8488	.9176	.9480	.9624
40	*	.3686	.5684	.7034	.7460	.8410
40	**	.8046	.9348	.9680	.9806	.9878
50	*	.4340	.6686	.8052	.8780	.9156
50	**	.9034	.9790	.9916	.9956	.9984

Powers for Testing H_o: Population is Uniform,
When Actual Population is Cauchy
Anderson-Darling Statistic

Maximum Likelihood Estimators						
n	Calculation Method *- standard **- reflected	Powers at Alpha Level .01	.05	.10	.15	.20
10	*	.3686	.4822	.5422	.5890	.6308
10	**	.3252	.5050	.5234	.5434	.5678
20	*	.6986	.8316	.8920	.9246	.9444
20	**	.4474	.6418	.7262	.7828	.8146
30	*	.9224	.9764	.9920	.9942	.9960
30	**	.7004	.8648	.9194	.9460	.9592
40	*	.9892	.9980	.9994	.9998	.9998
40	**	.8940	.9648	.9830	.9892	.9938
50	*	.9984	.9996	1.0000	1.0000	1.0000
50	**	.9714	.9934	.9966	.9988	.9994
Best Linear Unbiased Estimators						
n	Calculation Method *- standard **- reflected	Powers at Alpha Level .01	.05	.10	.15	.20
10	*	.3570	.5380	.6218	.7014	.7728
10	**	.1790	.3568	.4752	.5866	.6660
20	*	.7592	.9136	.9594	.9764	.9840
20	**	.5078	.7312	.8186	.8838	.9200
30	*	.9566	.9892	.9960	.9980	.9986
30	**	.7966	.9324	.9716	.9836	.9906
40	*	.9944	.9994	.9998	1.0000	1.0000
40	**	.9422	.9880	.9960	.9982	.9992
50	*	.9994	1.0000	1.0000	1.0000	1.0000
50	**	.9856	.9986	.9992	.9998	1.0000

Powers for Testing H_O: Population is Uniform,
When Actual Population is Triangular
Anderson-Darling Statistic

Maximum Likelihood Estimators

n	Calculation Method *- standard **- reflected	Powers at Alpha Level .01	.05	.10	.15	.20
10	*	.0046	.0248	.0504	.0784	.1084
10	**	.2506	.8602	.8644	.8686	.8720
20	*	.0056	.0280	.0592	.0964	.1302
20	**	.0566	.3590	.6398	.8916	.8968
30	*	.0046	.0302	.0790	.1292	.1942
30	**	.0554	.3686	.5930	.7808	.9046
40	*	.0072	.0624	.1514	.2354	.3318
40	**	.1418	.4858	.6924	.8118	.8882
50	*	.0130	.1048	.2652	.3952	.4984
50	**	.2896	.6154	.7908	.8836	.9310

Best Linear Unbiased Estimators

n	Calculation Method *- standard **- reflected	Powers at Alpha Level .01	.05	.10	.15	.20
10	*	.0024	.0220	.0534	.0906	.1404
10	**	.0416	.1912	.3112	.3972	.4634
20	*	.0090	.0692	.1620	.2564	.3676
20	**	.2164	.4748	.5858	.6764	.7336
30	*	.0234	.1428	.3556	.5094	.6242
30	**	.4136	.6878	.7946	.8486	.8896
40	*	.0518	.3180	.5816	.7302	.8158
40	**	.6266	.8534	.9164	.9426	.9588
50	*	.1036	.4942	.7670	.8798	.9254
50	**	.7912	.9350	.9666	.9802	.9880

Powers for Testing H_O: Population is Uniform,
When Actual Population is Double Exponential
Anderson-Darling Statistic

Maximum Likelihood Estimators

n	Calculation Method *- standard **- reflected	Powers at Alpha Level .01	.05	.10	.15	.20
10	*	.0744	.1676	.2354	.2964	.3494
10	**	.1208	.4004	.4082	.4162	.4294
20	*	.1840	.3558	.4932	.5864	.6638
20	**	.0770	.2572	.3996	.5298	.5824
30	*	.3716	.6386	.7992	.8640	.9100
30	**	.3142	.5750	.7014	.7894	.8368
40	*	.6376	.8860	.9534	.9754	.9862
40	**	.6400	.8156	.8754	.9082	.9328
50	*	.8452	.9690	.9928	.9966	.9978
50	**	.8240	.9196	.9522	.9706	.9804

Best Linear Unbiased Estimators

n	Calculation Method *- standard **- reflected	Powers at Alpha Level .01	.05	.10	.15	.20
10	*	.0400	.1554	.2342	.3404	.4608
10	**	.1204	.2844	.4430	.6132	.7044
20	*	.1908	.4318	.6206	.7234	.7994
20	**	.3058	.6574	.7710	.8430	.8822
30	*	.4000	.7034	.8614	.9204	.9518
30	**	.6446	.8596	.9186	.9466	.9602
40	*	.6556	.8968	.9614	.9814	.9918
40	**	.8380	.9452	.9712	.9824	.9898
50	*	.8232	.9686	.9918	.9972	.9982
50	**	.9264	.9804	.9896	.9952	.9972

Appendix F

Power Tables for the CVM Statistic

Powers for Testing H_0: Population is Uniform, When Actual Population is Normal
Cramer-Von Mises Statistic

Maximum Likelihood Estimators

n	Calculation Method *- standard **- reflected	Powers at Alpha Level .01	.05	.10	.15	.20
10	*	.0208	.0818	.1410	.1966	.2396
10	**	.0026	.0148	.0332	.0552	.0820
20	*	.0586	.1620	.2470	.3290	.3948
20	**	.0282	.0906	.1608	.2248	.2864
30	*	.1012	.2738	.4148	.5144	.6046
30	**	.1684	.3604	.4946	.5704	.6244
40	*	.1994	.4454	.6042	.7138	.7786
40	**	.4122	.6426	.7418	.8014	.8422
50	*	.3086	.6110	.7656	.8474	.8924
50	**	.6406	.8354	.8910	.9236	.9396

Best Linear Unbiased Estimators

n	Calculation Method *- standard **- reflected	Powers at Alpha Level .01	.05	.10	.15	.20
10	*	.1606	.2964	.3878	.4538	.5110
10	**	.2266	.4034	.5034	.5918	.6444
20	*	.2560	.4124	.5302	.6094	.6736
20	**	.3920	.6108	.7158	.7724	.8218
30	*	.3256	.5292	.6390	.7254	.7830
30	**	.6338	.7954	.8730	.9122	.9344
40	*	.4072	.6084	.7408	.8158	.8669
40	**	.7772	.9074	.9512	.9658	.9768
50	*	.4778	.6970	.8108	.8764	.9254
50	**	.8796	.9660	.9826	.9902	.9932

Powers for Testing H_O: Population is Uniform,
When Actual Population is Cauchy
Cramer-Von Mises Statistic

Maximum Likelihood Estimators						
	Calculation Method	Powers at Alpha Level				
n	*- standard **- reflected	.01	.05	.10	.15	.20
10	*	.4532	.5690	.6372	.6892	.7236
10	**	.2448	.3192	.3786	.4210	.4662
20	*	.7850	.8960	.9370	.9566	.9648
20	**	.5722	.6950	.7716	.8208	.8546
30	*	.9486	.9852	.9950	.9972	.9976
30	**	.8210	.9130	.9524	.9652	.9732
40	*	.9928	.9990	.9992	.9996	.9996
40	**	.9384	.9802	.9902	.9948	.9964
50	*	.9986	.9998	.9998	.9998	1.0000
50	**	.9838	.9958	.9988	.9992	.9996

Best Linear Unbiased Estimators						
	Calculation Method	Powers at Alpha Level				
n	*- standard **- reflected	.01	.05	.10	.15	.20
10	*	.4010	.5660	.6632	.7220	.7684
10	**	.0920	.2710	.3738	.4772	.5474
20	*	.8042	.9126	.9528	.9666	.9754
20	**	.5088	.7182	.8122	.8596	.8976
30	*	.9546	.9902	.9958	.9972	.9978
30	**	.8374	.9240	.9636	.9784	.9876
40	*	.9924	.9990	.9998	.9998	.9998
40	**	.9440	.9836	.9946	.9966	.9982
50	*	.9992	.9996	.9998	1.0000	1.0000
50	**	.9856	.9976	.9994	.9998	.9998

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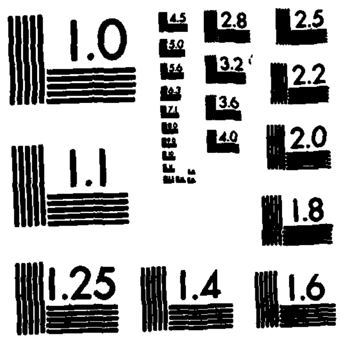

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Powers for Testing H_0: Population is Uniform,
When Actual Population is Cauchy
Cramer-Von Mises Statistic

Maximum Likelihood Estimators

n	Calculation Method *- standard **- reflected	Powers at Alpha Level .01	.05	.10	.15	.20
10	*	.4532	.5690	.6372	.6892	.7236
10	**	.2448	.3192	.3786	.4210	.4662
20	*	.7850	.8960	.9370	.9566	.9648
20	**	.5722	.6950	.7716	.8208	.8546
30	*	.9486	.9852	.9950	.9972	.9976
30	**	.8210	.9130	.9524	.9652	.9732
40	*	.9928	.9990	.9992	.9996	.9996
40	**	.9384	.9802	.9902	.9948	.9964
50	*	.9986	.9998	.9998	.9998	1.0000
50	**	.9838	.9958	.9988	.9992	.9996

Best Linear Unbiased Estimators

n	Calculation Method *- standard **- reflected	Powers at Alpha Level .01	.05	.10	.15	.20
10	*	.4010	.5660	.6632	.7220	.7684
10	**	.0920	.2710	.3738	.4772	.5474
20	*	.8042	.9126	.9528	.9666	.9754
20	**	.5088	.7182	.8122	.8596	.8976
30	*	.9546	.9902	.9958	.9972	.9978
30	**	.8374	.9240	.9636	.9784	.9876
40	*	.9924	.9990	.9998	.9998	.9998
40	**	.9440	.9836	.9946	.9966	.9982
50	*	.9992	.9996	.9998	1.0000	1.0000
50	**	.9856	.9976	.9994	.9998	.9998

Powers for Testing H_O: Population is Uniform, When Actual Population is Triangular
Cramer-Von Mises Statistic

Maximum Likelihood Estimators

n	Calculation Method *- standard **- reflected	Powers at Alpha Level .01	.05	.10	.15	.20
10	*	.0102	.0532	.1006	.1454	.1880
10	**	.0022	.0146	.0312	.0532	.0798
20	*	.0150	.0676	.1260	.1826	.2358
20	**	.0074	.0434	.0894	.1410	.1908
30	*	.0176	.0834	.1676	.2552	.3398
30	**	.0570	.1880	.3098	.3970	.4558
40	*	.0284	.1302	.2530	.3716	.4724
40	**	.1684	.4016	.5310	.6110	.6716
50	*	.0340	.1800	.3590	.5048	.6090
50	**	.3188	.6056	.7252	.7914	.8318

Best Linear Unbiased Estimators

n	Calculation Method *- standard **- reflected	Powers at Alpha Level .01	.05	.10	.15	.20
10	*	.0256	.1156	.2214	.3148	.4044
10	**	.1264	.3776	.5096	.6174	.6838
20	*	.0410	.1854	.3628	.4954	.6084
20	**	.3628	.6492	.7652	.8191	.8582
30	*	.0546	.2900	.5032	.6642	.7636
30	**	.6278	.8138	.8872	.9230	.9418
40	*	.0950	.4196	.6686	.7936	.8682
40	**	.7566	.9098	.9514	.9690	.9794
50	*	.1742	.5680	.1802	.8752	.9360
50	**	.8654	.9640	.9846	.9906	.9940

Powers for Testing H_O: Population is Uniform,
When Actual Population is Double Exponential
Cramer-Von Mises Statistic

Maximum Likelihood Estimators

n	Calculation Method *- standard **- reflected	Powers at Alpha Level .01	.05	.10	.15	.20
10	*	.0930	.2110	.2884	.3546	.4052
10	**	.0074	.0294	.0562	.0938	.1346
20	*	.2372	.4458	.5740	.6626	.7264
20	**	.1418	.2868	.3896	.4696	.5288
30	*	.4232	.7006	.8288	.8934	.9282
30	**	.4638	.6472	.7390	.7908	.8206
40	*	.6812	.8910	.9522	.9756	.9862
40	**	.7204	.8454	.8874	.9170	.9330
50	*	.8440	.9730	.9900	.9960	.9976
50	**	.8502	.9278	.9540	.9674	.9740

Best Linear Unbiased Estimators

n	Calculation Method *- standard **- reflected	Powers at Alpha Level .01	.05	.10	.15	.20
10	*	.1424	.3004	.4058	.4904	.5538
10	**	.1188	.2476	.3526	.4638	.5368
20	*	.3296	.5476	.6872	.7764	.8318
20	**	.2566	.5610	.6976	.7696	.8178
30	*	.5040	.7814	.8738	.9254	.9500
30	**	.6390	.8142	.8844	.9204	.9376
40	*	.7136	.9124	.9648	.9808	.9882
40	**	.8156	.9210	.9548	.9690	.9786
50	*	.8690	.9760	.9910	.9948	.9974
50	**	.9076	.9664	.9820	.9894	.9930

Vita

Larry Bruce Woodbury was born on 1 October 1950, in St. George, Utah, to Marvin H. and Dora Jean Woodbury. He attended elementary schools in Phoenix, Arizona and in St. George, Utah. He also attended Dixie High School in St. George where he graduated in 1968. After attending one year at Dixie Jr. College, he served as a missionary for the LDS Church for two years in Argentina. Upon returning home in November 1971, he again attended Dixie Jr. College for another year, after which he transferred to Southern Utah State College in Cedar City, Utah. While at S.U.S.C. he joined the Air Force R.O.T.C. on the two-year program. He received a Bachelor of Science degree in Mathematics and a commission in the U.S. Air Force at the same time in May 1974. Captain Woodbury attended undergraduate pilot training at Williams AFB, Arizona where he earned his wings in January 1976. His follow-on assignment was to the 53rd Military Airlift Squadron (MAC) at Norton AFB, California where he served as co-pilot and Aircraft Commander in the C-141 aircraft. In May 1981, Captain Woodbury was assigned to the School of Engineering, Operations Research Department, at the Air Force Institute of Technology.

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POWER STUDY	MAXIMUM LIKELIHOOD ESTIMATORS

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Separate techniques of interpolation, reflection, and parameter estimation are combined to develop a new goodness of fit test for the uniform distribution. The Kolmogorov-Smirnov, Anderson-Darling, and Cramer-Von Mises statistics are used in the generation of critical value tables of sample sizes from 3 to 50. The methods for estimating parameters are the Maximum Likelihood and the Best

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Linear Unbiased Estimators. Separate tables for each are presented. These tables are built with and without employing the reflection technique. The reflection technique is one in which the data points are reflected about the sample mean to double the size of the sample set.

With these tables of critical values, a power study is done to test the power of the three statistics with the reflection procedure versus the same statistics without the reflection procedure. The powers are generally higher for the statistics modified with the reflection procedure; however, they are found to be smaller for data distributions that are non-symmetrical or Cauchy. The power for the Anderson-Darling statistic using the Maximum Likelihood Estimators is found to be of little value while the powers of all statistics were found to be improved by using the Best Linear Unbiased Estimators instead of the Maximum Likelihood Estimators.